Enhanced performance of microfluidic soft pressure sensors with embedded solid microspheres
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Abstract
The cross-sectional geometry of an embedded microchannel influences the electromechanical response of a soft microfluidic sensor to applied surface pressure. When a pressure is exerted on the surface of the sensor deforming the soft structure, the cross-sectional area of the embedded channel filled with a conductive fluid decreases, increasing the channel’s electrical resistance. This electromechanical coupling can be tuned by adding solid microspheres into the channel. In order to determine the influence of microspheres, we use both analytic and computational methods to predict the pressure responses of soft microfluidic sensors with two different channel cross-sections: a square and an equilateral triangular. The analytical models were derived from contact mechanics in which microspheres were regarded as spherical indenters, and finite element analysis (FEA) was used for simulation. For experimental validation, sensor samples with the two different channel cross-sections were prepared and tested. For comparison, the sensor samples were tested both with and without microspheres. All three results from the analytical models, the FEA simulations, and the experiments showed reasonable agreement confirming that the multi-material soft structure significantly improved its pressure response in terms of both linearity and sensitivity. The embedded solid particles enhanced the performance of soft sensors while maintaining their flexible and stretchable mechanical characteristic. We also provide analytical and experimental analyses of hysteresis of microfluidic soft sensors considering a resistive force to the shape recovery of the polymer structure by the embedded viscous fluid.

Keywords: soft sensors, stretchable electronics, microsphere, liquid metal, microfluidics

(Some figures may appear in colour only in the online journal)
[14, 15], curvature sensors for flexible structures [16, 17], wearable soft sensors for body motion sensing [18–20], and a tactile sensing skin for force localization and feedback [21, 22].

For this class of soft sensors, the electrical resistance of the embedded microchannels is highly dependent on the deformation of the surrounding elastomer caused by external mechanical stresses [11]. Based on its elastic deformation, the soft sensor can detect different deformation modes such as compression or strain. The resistance change of the microchannel is directly related to the changes of the microchannels’ cross-section and length, caused by the applied stress. In this case, the resistivity of the liquid conductor can be assumed to remain constant since the mechanical stress applied to the liquid conductor itself is insignificant. Therefore, the geometrical change of the microchannels has a dominant influence on the sensor response while the piezoresistive effect of the liquid conductor is negligible. For strain sensing, gauge factors of 3.4–3.6 with microchannels of 0.060–0.063 mm² (cross-sectional area) have been reported [14, 23]. In case of pressure sensing, the effect of the cross-sectional geometry of a microchannel has been investigated in previous research, confirming that the pressure response, in terms of sensitivity and linearity, could be controlled by the shape of the microchannel [24, 25]. However, such methods involve a specialized micro-machining process to produce a microchannel mold that includes sharp features with high accuracy, for elastomer casting. Moreover, the reliability of the molds may be compromised from multiple uses of the mold due to the sharp and fragile edge of the microchannel feature. Alternatively, we show that electromechanical coupling can instead be controlled by the addition of solid microspheres in a typical rectangular microchannel [26]. The proposed method demonstrates not only a more desirable, linearized pressure response, but also a significant reduction in the complexity of manufacturing. Furthermore, the discrete characteristic of the microspheres maintains the same stretchability and flexibility of the soft sensors while allowing us to utilize their solid property.

In this paper, we introduce a soft pressure sensor composed of soft silicone elastomer embedded with microfluidic channels of EGaIn that contain solid microspheres. We examine electromechanical coupling for two different channel cross-sections—square and triangle—using analytical modeling, finite element analysis (FEA), and experiments. The theoretical models were derived by correlating hyperelastic deformation with contact mechanics. Since the theory depends only on the geometries of the microchannels and microspheres, it is generalized from a part of cross-section of the microchannel under an assumption of geometrical symmetry. The predictions from both analytical modeling and FEA simulation showed reasonable agreement with the experimental measurements.

2. Fabrication

Figure 1 shows four different microchannel geometries that we examined. Two have square cross-sections and the other two have equilateral triangular cross-sections, with and without microspheres (316SS—stainless steel balls, Salem Specialty Ball Co.). All four channels have the same cross-sectional area of 0.25 mm². The microchannels are embedded in silicone elastomer (EcoFlex30, Smooth-On) and filled with a liquid-phase eutectic gallium-indium alloy (EGaIn, \( \rho = 29.8 \times 10^{-8} \) \( \Omega \) m\(^{-1} \)). Figure 2 shows microscopic images of the microchannels and embedded microspheres. The samples were fabricated by casting two elastomer layers, one with a microchannel pattern and the other with a flat surface, using micromachined plastic molds. After the two layers were cured, the microspheres were manually inserted in the microchannel with an open top before bonding the flat layer. During insertion, the microspheres were aligned so they could be in contact not only with each other but also with the surrounding walls of the microchannel. Then, the flat layer was laminated on top of the microchannel layer. Before lamination, the same uncured silicone should be spin-coated on the surface of the flat layer. The spin-coated liquid silicone not only works as an adhesive layer between the two cured layers by making cross-links as it cures but also holds the microspheres at their original positions by adhering to them even though the completed sample experiences deformation. Finally, EGaIn was injected into the microchannel. The high surface tension of EGaIn facilitated expelling the air captured during the bonding process from the microchannel leaving no air bubbles inside. More details on fabrication can be found in some of our previous work [11, 14].

![Figure 1. Dimensions of the sensor samples used in experiments with four different cross-sectional geometries: (a) empty square, (b) microsphere embedded square, (c) empty triangle, and (d) microsphere embedded triangle.](image-url)
3. Theory

For the analytic model, we treat the microsphere as a rigid body indenter that presses against the surrounding walls of the microchannel, as shown in figures 3 and 4. Also, we treat the walls as an elastic half-space so that we can adopt existing solutions from contact mechanics [27]. When a pressure, \( p \), is applied to the surface of the elastomer, it induces an internal stress of a vertical component \( \sigma_z \) that can be approximated as \( \chi p \), where \( \chi = \chi(x, z) \) is obtained from Boussinesq’s method [11]. Then, a point load applied on each microsphere can be expressed as \( F = \sigma \pi r_0^2 \).

Since the elastomer used in our sensor is subject to finite strain, we treat it as a Neo-Hookean solid. Based on the solution [28] derived from the definitions of indentation stress and strain [29] and the hyperelastic strain energy function [30], the relationship between \( F \) and a contact radius \( a_0 \) can be expressed as

\[
F = B_1 \pi a_0^5 - 15r_0 a_0^4 + 75r_0^2 a_0^3 - 50r_0^3 a_0 + 125r_0^5
\]

(1)

where \( r_0 \) is the radius of the microsphere, \( B_1 \) is the neo-Hookean coefficient defined as

\[
B_1 = \frac{20E}{9\pi(1-\nu^2)},
\]

(2)

and \( E \) and \( \nu \) are the Young’s modulus (125 kPa) and the Poisson’s ratio (0.49) of the elastomer, respectively [11].

Next, we find \( a_0 \) using \( p \), \( E_0 \), and \( \nu \), in (1). An indentation \( \delta = r_0 - \sqrt{r_0^2 - a_0^2} \) is calculated from the obtained \( a_0 \) and the geometries, shown in figures 3 and 4, instead of using the relationship between \( a \) and \( \delta \) from the Herzian contact [27] or from a fitting function [28]. Since the radius of the cross-section of the microsphere changes along the longitudinal direction of the microchannel, the radius \( r \) and the contact radius \( a \) need to be obtained first. From the geometries, we can express \( r \) and \( a \) as functions of \( y \), such as \( r(y) = \sqrt{r_0^2 - y^2} \) and \( a(y) = \sqrt{r^2 - (r_0 - \delta)^2} \).

Figure 2. Microscopic images of cross-sections of microsphere embedded microchannels with (a) a square and (b) an equilateral triangular shape.

Figure 3. Cross-sectional geometries and parameters used in analytical models for a square microchannel with embedded microspheres in contact (0 ≤ y ≤ a_0) and not in contact (a_0 < y ≤ r_0) with channel surfaces.
3.1. Area calculation of square channel

In case of square microchannels, the quarter space of the microchannel cross-section with a microsphere is used, as shown in figure 3, assuming symmetry of cross-sectional deformation. In this case, the displacement of the side wall in x-axis was not taken into account assuming the deformation in x-axis is negligible. Based on \( a_0 \), the regions along the y-axis can be divided into two cases: in contact \((0 \leq y \leq a_0)\) and not in contact \((a_0 \leq y \leq r_0)\), as shown in figure 3. The cross-sectional area, \( A_1(y) \), of the regions in contact can be approximated as

\[
A_1(y) = 4 \left( r_0(r_0 - \delta) - \frac{1}{2} \left[ a(r_0 - \delta) + r^2 \theta \right] \right)
\]

where the angle \( \theta(y) = \arcsin((r_0 - \delta)/r) \). Also, the cross-sectional area not in contact can be expressed as

\[
A_2(y) = 4 \left\{ r_0(r_0 - \delta) - \frac{\pi r^2}{4} \right\}.
\]

3.2. Area calculation of triangular channel

In the triangular channel case, a one-sixth portion of the microchannel with a microsphere is examined in the model. Similar to the square channels, the regions are also divided into the ones in contact and not in contact. The cross-sectional area of the deformed region in contact, in figure 4, can be approximated as

\[
A_1(y) = \sqrt{s(s-l)(s-m)(s-n)} + \frac{1}{2} (r_0 - \delta) (a_0 - a) - \frac{1}{2} r^2 \theta,
\]

where \( s = (l + m + n)/2 \) in Heron’s formula for area calculation of a triangle, \( l = \sqrt{(w - a_0) + \delta^2}, m = r_0, n = w/\cos(\pi/6) \), and \( \theta = \pi/3 - \arctan(a/(r_0 - \delta)) \).

Similarly, the area of the deformed regions not in contact can be approximated as

\[
A_2(y) = \sqrt{s(s-l)(s-m)(s-n)} + \frac{1}{2} a_0(r_0 - \delta) - \frac{\pi r^2}{6}.
\]

Since the tractions applied on the side walls are half of those on the bottom wall, we need to distinguish the case of the side walls from that of the bottom wall and to apply a point load, \( F/2 \), to the side walls. In addition, the area should be calculated as \( A = 2A_{\text{bottom}} + 4A_{\text{side}} \) in both \( A_1 \) and \( A_2 \) since there are two sections on the bottom of the triangular channel and four sections on the sides.

3.3. Electrical resistance calculation

For both cases of square and triangular channels, we can obtain the electrical resistance by numerically integrating \( A_1 \) and \( A_2 \), along the y-axis:

\[
R(y) = \int_{0}^{a_0} \frac{\rho}{A_1} \, dy + \int_{a_0}^{r_0} \frac{\rho}{A_2} \, dy.
\]

4. Simulation

We implemented a series of three-dimensional (3D) stress simulations for predicting the deformation of an incompressible silicone rubber \((E = 125 \text{ kPa} \text{ and } \nu = 0.49)\) affected by embedded microspheres \((E = 205 \text{ GPa} \text{ and } \nu = 0.275)\) using a commercial FEA software package (ABAQUS/CAE 6.13-3, Dassault System). In order to reduce the process time, we reduced the size of the models, as shown in figure 5. For boundary conditions, the surfaces of the model in the bottom and left end were fixed and the left end surface with the microspheres was allowed to move only in y-axis. The other parts were fixed only in z-axis. In addition to the boundary conditions, a pressure with linear increments (0 kPa to 70 kPa) was applied on the top surface with a fixed width (12.5 mm). Frictionless contact condition was applied to both the contacts of the microchannel itself (self-contact) and the contacts between the channel walls and microspheres (surface contact). A neo-Hookean model was used with coefficients \( C_{10} = 0.02097 \) and \( D_1 = 0.96 \) in ABAQUS/CAE [31]. These coefficients were found using the bulk modulus of the silicone rubber, which was obtained by Lamé parameters [32] with given \( E \) and \( \nu \). Based on the coordinates of the deformed microchannel, the estimated area changes of the microchannels calculated in post-processes were used to evaluate the changes of the electrical resistance.

As shown in figure 5, the shape of each corner of the microchannel with embedded microspheres is similar to that of a
5. Results

Figure 6 shows the overall experimental setup for measuring pressure and resistance changes simultaneously. Each sample was tested by applying a normal pressure at the center of the top surface using a flat square indenter (25 mm × 25 mm) up to 70 kPa, as shown in figures 6(b) and (c). A load was gradually increased on this pressure area at a rate of 1 mm s⁻¹ using a motorized materials test stand (ESM301, Mark-10) and simultaneously measured using a commercial single-axis load cell (STL-50, AmCells). For sensor output reading, a series circuit of a 10Ω resistor and a current source of 100 mA was constructed. The voltage output was obtained by a data acquisition software (LabVIEW 2010, National Instruments Corp.) through a data acquisition board (NI myDAQ, National Instrument Corp.) and then converted to the ratio of resistance change ∆R/R₀. The data were collected at a sampling rate of 100 Hz.

Figure 7 shows the experimental results of the microsphere embedded channels (solid blue lines) and empty channels (solid red lines) for both square figure 7(a) and triangular figure 7(b) cross-sections. The theoretical and FEA predictions, in the same plots, showed reasonable agreements with the experimental results, confirming the significant improvements in linearity, sensitivity at low pressure, and also in dynamic ranges of the signals. The differences between the experimental results and the predictions are mainly from the inaccurate material properties for modeling and simulation and also from the manufacturing tolerances in the experiments. The dotted blue lines show the resistance changes evaluated from equation (7), based on the same dimensions of the prepared samples and an applied pressure p ranging from 0 kPa to 70 kPa. The FEA simulation results were shown with dashed blue lines in the same figure.

Taking advantage of the geometrical benefit, the microchannels with microspheres showed more of a linear response than regular empty channels, resulting in higher sensitivities at low pressure and larger dynamic ranges.

6. Hysteresis analysis

It is well known that soft structures show relatively large hysteresis when returning to their original shapes after experiencing deformation even though the external force was removed. This is mainly due to the viscoelastic effect of the base polymer. Such hysteresis has been previously reported in soft microfluidic sensors [14, 24, 33] and also in our sensor samples, as shown in figure 8, since the viscoelasticity of the polymer causes a time delay for the conductive fluid to physically move in the microchannel even after the release of deformation (i.e. pressure). However, the recovery of the channel shape by the elastic force is also resisted by the viscous fluid within the channel. This aspect can be addressed by employing the hydraulic diameter Dh = 4A/p, where A is the cross sectional...
area and $P$ is the wetted perimeter. For laminar flow, the role of the wall friction is more significant for smaller $D_h$. Without beads, the hydraulic diameters are similar for both channels. They are $D_h = d = 0.50$ and $D_h = \frac{2l}{3} = 0.51$ for square and triangular channels, respectively, where $l$ is the length of one side of the triangular channel. However, $D_h$ becomes very different for these channels with microspheres. At the cross-section where microspheres make contacts with the channel walls, $D_h = \frac{4 - \tau - d}{4 + \tau} = 0.06$ and $D_h = \frac{\sqrt{3}l^2 - \pi d^2}{3l + \pi d} = 0.11$ are estimated for square and triangular channels, respectively. Wetted perimeters were calculated by summing the perimeters of the channel wall and the microsphere surface at the cross-section. Note that the square channel shows significantly smaller $D_h$. This indicates a higher resistive force from the fluid for the square channel with microspheres than the triangular channel, which explains the results in figure 8.

Such resistive force exerted by a viscous fluid in the channel can be solved analytically for an ideal two-dimensional (2D) channel. There have been extensive analytical and numerical studies on the flow distribution during expansion or contraction of microchannels, with [34, 35] and without [36] permeability on the walls. We adopted analytical formulations from [34] and calculated the velocity and the pressure distribution with zero permeability. Figure 9(a) demonstrates the velocity vectors and stream lines during expansion of a 2D channel. $\eta$ and $\zeta$ represent the horizontal and vertical directions, respectively. $\tau$ and $\tau$ are the half of the channel thickness (i.e. channel walls are at $\zeta/\tau = -1$ and 1). Analytical studies typically assume that there is a membrane at $\eta = 0$ that does not affect the vertical flow but maintains symmetry of the flow at $\eta = 0$. The pressure distribution in the vertical direction is given by

$$\frac{P(\eta, \zeta) - P(\eta, \zeta = 0)}{\rho v^2_w} = -0.5G^2 + \alpha^2G + \alpha^{-1}G$$

where $\alpha = \tau lv$, $\tau$ and $\tau$ are the half of the channel thickness and the velocity of the wall, respectively, and $G$ and $G$ are the similarity solutions for the problem that can be found in [34]. Figure 9(b) shows the pressure distribution in the vertical direction for EGaIn, liquid mercury, and water, normalized by the pressure at the wall for EGaIn. Due to smaller viscosity, negative wall pressure of water is $\approx 50\%$ of the value for EGaIn. This suggests that if a water-based ionic liquid is used, as proposed in [6], the recovery of the channel becomes faster than liquid mercury, and the performance of the liquid sensor may be improved.

7. Discussion

Our results demonstrated that inserting solid microspheres into the microfluidic channel of a soft sensor improved the sensor performance with a more linear and sensitive pressure response. The main advantage of this method is incorporation of the solid mechanical property of the microspheres in a soft structure while not only maintaining the structure’s highly stretchable and flexible characteristic but also keeping the manufacturing process simple. However, there are a few challenges that need to be addressed for our proposed method to be more practical.
The current fabrication method is also another challenge since it does not always guarantee complete filling of the liquid metal in the microchannel. When a flat layer is laminated on the microchannel layer with microspheres, part of the uncured thin silicone coating, used as a bonding agent, sometimes flows into the small gaps between the microspheres and the channel walls, resulting in blockages in the microchannel, as shown in figure 10. This reduces the amount of the liquid metal to be injected, increasing the resistance in an unpredictable way. Therefore, this irregular microchannel could not only be a source of an inconsistent sensitivity of the sensor but also cause increase in the hysteresis level. One possible solution is to find a better way of controlling the thickness of the uncured silicone coating [37], which will minimize the amount of silicone necessary for bonding. There have been various techniques proposed for bonding PDMS layers, such as oxygen plasma bonding [38], stamp and stick (SAS) transfer bonding [39], and corona discharge [40]. Further investigation on more efficient bonding processes based on the above techniques will be one of our immediate future research areas.

Figure 7. Pressure responses of (a) square and (b) equilateral triangular channels without (red) and with (blue) microspheres. Color bands denote the standard deviation of five trials. Theoretical predictions and FEA simulation results of microsphere embedded channels are shown with dotted lines and dashed lines, respectively.

Figure 8. Experimental results of hysteresis analysis for a square (blue) and a triangular (red) microchannel with pressure loading-unloading loops. The triangular channel shows slightly less hysteresis due to the larger spaces in the channel that facilitate the flow of the liquid conductor.

Figure 9. (a) Stream lines and velocity vectors during the expansion of a two-dimensional channel. (b) Pressure distribution in the vertical direction for EGaIn (red solid), liquid mercury (blue dashed) and water (black dotdash), normalized by the pressure at the wall for EGaIn.
In particular, adding the microspheres led to an increase in the electromechanical response to applied surface pressure. Microspheres in a microchannel had a significant influence on models, simulations, and experiments. The results from the theoretical models were validated with both FEA simulations and experimental measurements. The results from the models, simulations, and experiments confirmed that adding microspheres in a microchannel had a significant influence on the electromechanical response to applied surface pressure. In particular, adding the microspheres led to an increase in linearity, sensitivity, and dynamic range. Although the experimental results showed some hysteresis in the sensor response, this could potentially be reduced by changing the number or shape of the embedded microspheres. The theory could also be improved to model sensor hysteresis caused by friction between the fluid, microspheres, and channel walls.

Another contribution is the analytical approach to explain the hysteresis of microfluidic soft sensors based on fluid mechanics. Hysteresis has been analyzed only experimentally and discussed only based on the viscoelasticity of the base polymer in previous work. However, we discovered another important factor that affects the hysteresis: a resistive force to the recovery of polymer by a viscous fluid. The result analytically proved that hysteresis can be alleviated or more severe if different types of liquid conductors are used.

Once the fabrication technique is mature enough to make the sensor response reliable with less hysteresis, the proposed method will also have a potential for commercialization since soft sensors are one of the enabling technologies for future robots and robotic products, such as wearable robots, soft robots, medical devices, and haptic devices. Furthermore, the linear response will make the soft sensor technology more easily applicable to various sensor products that are already in the market.

8. Conclusion

The main contribution of this work is to introduce and examine soft sensor architectures composed of solid microspheres embedded in a microchannel filled with a conductive liquid, and their analytical and experimental characterizations. The theoretical models were validated with both FEA simulations and experimental measurements. The results from the models, simulations, and experiments confirmed that adding microspheres in a microchannel had a significant influence on the electromechanical response to applied surface pressure.

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