
ELSEVIER
Available online at www.sciencedirect.com
SciVerse ScienceDirect
MECHANICS
RESEARCH COMMUNICATIONS
An International Journal for the Rapid Communication
of Basic and Applied Contributions in Mechanics
of Fluids, Solids, Particles, and Systems
Sponsored by CISM
International Centre for Mechanical Sciences

Mechanics Research Communications 49 (2013) 13–16

Contents lists available at SciVerse ScienceDirect

Mechanics Research Communications

journal homepage: www.elsevier.com/locate/mechrescom

Bifurcations and instability in the adhesion of intrinsically curved rods

ARTICLE INFO

Keywords:
Adhesion
Rod theory
Stability
Elastica

ABSTRACT

Motivated by applications such as gecko-inspired adhesives and microdevices featuring slender rod-like bodies, there has been an increase in interest in the deformed shapes of elastic rods adhering to rigid surfaces. A central issue in analyses of the rod-based models for these systems is the stability of the predicted equilibrium configurations. Such analyses can be complicated by the presence of intrinsic curvatures induced by fabrication processes. The results in the present paper are used to show how this curvature can lead to shear-induced bifurcations and instabilities. To characterize potential instabilities, a new set of necessary conditions for stability are employed which cater to the possible combinations of buckling and delaminating instabilities.

1. Introduction

In emerging fields like stretchable electronics and micro-opto-electromechanical systems (MOEMS), deposited thin films may be designed to simultaneously buckling and delaminate from pre-stretched elastomer substrates to form wavy structures (Qu et al., 2011; Rogers et al., 2010; Shih et al., 2008). Just as in other classes of flexible elastic structures, peeling may be studied with an elastic rod theory that treats the edge of the bonded contact zone as a free boundary governed by flexural rigidity, internal bending moment, and work of adhesion. In recent years, this approach has been used to model microelectronic switches (Adams and McGruer, 2010), stiction in MEMS devices (de Boer and Michalske, 1999), soft lithography stamp printing (Hui et al., 2002), muscle crossbridges (Stewart et al., 1987), nanotubes (Glassmaker and Hui, 2004), and gecko-inspired microfiber array adhesives (Majidi, 2009).

A central issue for the solutions presented in many of the aforementioned studies is the paucity of analytical criteria for instability. In addition to the classical buckling-type instabilities experienced by rods, it is natural to ask if the adhesion boundary condition can become unstable to small perturbations? Such an instability would lead to delamination. Recent work by the authors (Majidi et al., 2012), which leveraged the existing formulations of peeling problems in Maddalena and Percivale (2008), Majidi (2009), O'Reilly (2007) and Plaut et al. (2001, 2001), established a set of criteria for problems featuring elastic rods with adhesion. Their criteria could be decomposed into one criterion for buckling and a second closely related criterion for adhesion instability. However, they found no examples which violated the latter criterion. In the present paper, we find that the presence of intrinsic curvature can produce adhesive instability (delamination). The results are directly applicable to many of the aforementioned devices where intrinsic curvature is a consequence of fabrication.

The boundary-value problem governing the static configurations is determined using a variational principle featuring the potential energy Π of the elastic rod. Exploiting classical methods by Legendre and Jacobi, an examination of the second variation of Π is used to establish necessary conditions, which we label N1, for the stability of the adhered rod. The criterion is based on an extension to our earlier works in Majidi et al. (2012) and O'Reilly and Peters (2012) and can be related to treatments of nonlinear buckling involving Jacobi's equation and the search for conjugate points such as Jin and Bao (2008), Kuznetsov and Levyakov (2002), Maddocks (1984), Manning (2009), Manning et al. (1998) and O'Reilly and Peters (2011) and references therein. The conditions N1 are then used to classify the configurations of a terminally loaded elastica which possesses an intrinsic curvature. Some of these configurations are unstable because any perturbation to the detachment point $\xi = \gamma$ will not decay.

2. The boundary-value problem and instability criteria

We start our analysis by using Euler's theory of the elastica to model the adhesion of a flexible, inextensible elastic rod $\mathcal{R}$ of length ℓ to a rigid surface. Referring to Fig. 1, the arclength of the centerline of the rod is parameterized using a coordinate ξ, and the position vector $\mathbf{r}$ of a material point labeled $\xi \in [0, \ell]$ has the representation $\mathbf{r} = X\mathbf{E}_1 + Y\mathbf{E}_2$. The unit tangent vector to the centerline of $\mathcal{R}$ has the representation $\mathbf{r}' = \cos(\theta)\mathbf{E}_1 + \sin(\theta)\mathbf{E}_2$, where the prime denotes the partial derivative with respect to ξ. The curvature κ of the centerline of $\mathcal{R}$ is $\kappa = \theta'$. When the centerline is unloaded, it relaxes into a curve with an intrinsic curvature κ_g.

For the problem at hand, discontinuities occur and it is necessary to define the following limits for any function $\chi = \chi\left(\xi, \theta, \theta'\right)$:

$$\chi(\zeta^-) = \lim_{\xi \nearrow \zeta} \chi(\xi, \theta(\xi), \theta'(\xi)),$$
$$\chi(\zeta^+) = \lim_{\xi \searrow \zeta} \chi(\xi, \theta(\xi), \theta'(\xi)). \tag{1}$$

0093-6413/$ – see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.mechrescom.2013.01.004

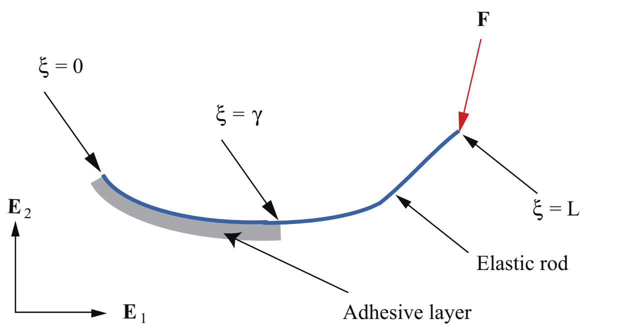

Fig. 1. Schematic of a terminally loaded rod of length ℓ. A portion of the rod of length γ is bonded to a fixed surface.

The jump in a function χ at $\xi = \chi$ is

$$[\![\chi]\!]_\zeta = \chi(\zeta^+) - \chi(\zeta^-). \tag{2}$$

Continuity of $\mathbf{r}$ and θ implies that $[\![\mathbf{r}']\!]_\xi = \mathbf{0}$ for all $\xi \in [0, \ell]$.

The bending moment $\mathbf{M}$ in the rod is prescribed by the classic constitutive equation $\mathbf{M} = D(\kappa - \kappa_g)\mathbf{E}_3$, where D is the flexural rigidity. We also introduce the contact force $\mathbf{n}$ acting on the rod. Assuming that a terminal load $\mathbf{F}$ acts at the tip $\xi = \ell$, then, from a balance of linear momentum, we find that $\mathbf{n}$ is a piecewise constant with $\mathbf{n}(\ell^-) = \mathbf{F}$. Modulo an additive constant, the total potential energy Π is

$$\Pi = \int_\gamma^\ell \left\{ \frac{D}{2}(\kappa - \kappa_g)^2 - \mathbf{n}\cdot\mathbf{r}' \right\} d\xi + \int_0^\gamma \left\{ \frac{D}{2}(\kappa - \kappa_g)^2 - \mathbf{n}\cdot\mathbf{r}' - \omega \right\} d\xi. \tag{3}$$

In this expression for Π, ω is the adhesive energy per unit contact length.

The first variation of Π leads to the boundary-value problem which is used to determine the deformed shape $\theta^*(\xi)$ of the rod for $\xi \in (\gamma, \ell)$:

$$(D(\kappa^* - \kappa_g))' - P_1 = 0, \tag{4}$$

where

$$P_1 = \mathbf{F}\cdot(\sin(\theta^*)\mathbf{E}_1 - \cos(\theta^*)\mathbf{E}_2), \qquad \kappa^* = \theta^{*\prime}(\xi). \tag{5}$$

Assuming that $\theta(\xi)$ is continuous and that the slope at the edge of the contact surface is $\tan(\theta_0)$, the sought-after solution θ^* to (4) needs to satisfy the following set of boundary conditions:

$$\theta^*(\gamma^+) = \theta_0, \qquad \theta^{*\prime}(\ell) = \kappa_g(\ell),$$

$$\omega + [\![\frac{D}{2}(\kappa^* - \kappa_g)^2 - \mathbf{n}\cdot\mathbf{r}']\!]_\gamma = (D(\kappa^* - \kappa_g))(\gamma^+)[\![\kappa^*]\!]_\gamma. \tag{6}$$

The second of these boundary conditions follows from the fact that there is no applied moment at the tip $\xi = \ell$, while the third condition is the adhesion boundary condition at $\xi = \gamma$. The presence of $\mathbf{n}\cdot\mathbf{r}'$ in $(6)_3$ makes it different from the adhesion boundary condition in other works – such as Eq. (6) in Majidi and Adams (2010) – but this difference is not an issue in the forthcoming examples such as those shown in Fig. 2. Fig. 2 presents (a) the natural shape of the elastic rod in the absence of adhesion, (b) the equilibrium deformation of the rod adhering to a substrate ($\mathbf{F} = 0\mathbf{E}_1$), and (c) various solutions for a partially adhering rod under an axial load $\mathbf{F} = -27\mathbf{E}_1$.

The stability criteria we employ are based on computing the second variation of the potential energy Π of the rod and can be considered as an extension to classical works on this topic which feature Jacobi's necessary condition. Using the arguments presented in Majidi et al. (2012), we can readily establish a necessary condition for stability, which is referred to as N1. The only changes needed are to include the intrinsic curvature κ_g and to relax the assumption in Majidi et al. (2012) that $\theta'(\gamma^-) = 0$. Here, we merely quote the criterion as it pertains to the rod-based model that we are examining and refer the reader to Majidi et al. (2012) for relevant background.

Condition N1: *If a solution* $\{\theta^*(\xi), \gamma^*\}$ *to the boundary-value problem minimizes* Π *then the solution* $w(\xi)\,\forall\xi \in (\gamma^*, \ell]$ *to the boundary-value problem*

$$\frac{\partial w}{\partial \xi} + P - \frac{w^2}{D} = 0, \tag{7}$$

with $w(\ell) = 0$ *and* $P = \mathbf{F}\cdot(\cos(\theta^*)\mathbf{E}_1 + \sin(\theta^*)\mathbf{E}_2)$ *cannot become unbounded in the interval* $[\gamma, \ell]$ *and the following inequality must be satisfied:*

$$\Gamma \geq 0. \tag{8}$$

The function Γ *depends on the solution* w *to* (7) *and the solution to the boundary value problem for* θ:

$$\Gamma = -[\![P_1\kappa^* + D(\kappa^{*\prime} - \kappa_g')(\kappa^* - \kappa_g)]\!]_\gamma + 2P_1(\gamma^+)[\![\kappa^*]\!]_\gamma + D[\![\kappa^{*\prime}]\!]_\gamma(\kappa^*(\gamma^+) - \kappa_g(\gamma^+)) + w(\gamma^+)[\![\kappa^*]\!]_\gamma^2. \tag{9}$$

The condition N1 is a necessary condition for the second variation of Π to be positive. It is well known that the existence of a bounded solution $w(\xi)$ to the Riccati equation (7) implies that the rod has not buckled but this criterion is silent on the state of the adhesive. However, as the adhesion boundary condition (6) can often be used to express $\theta^{*\prime}(\gamma^+)$ in terms of ω, we can interpret (8) as a stability condition for the adhesive. Thus N1 can be considered as a two-part criterion. First, existence of w implies that the equilibrium configuration of the rod satisfies a necessary condition for the minimization of Π with respect to perturbations that preserve γ^*. The second part of the criterion states that perturbing γ from its equilibrium value γ^* does not violate a necessary condition for the minimization of Π. If an equilibrium configuration does not satisfy *N*1 either because a bounded solution w cannot be found or $\Gamma < 0$, then the configuration is said to be unstable.

3. Adhesive instabilities

We are now in a position to examine the configurations of an intrinsically curved elastica and use the N1 condition to classify its stability. Thus, we consider an elastic rod that has intrinsic curvature κ_g and is loaded with an axial force $\mathbf{F} = -F\mathbf{E}_1$. The force $\mathbf{F}$ in this case is often classified as a shear force. Specifically, we consider a rod which has the following intrinsic shape and curvature:

$$\theta_g = \frac{2\pi A}{\ell}\sin\left(\frac{2\pi\xi}{\ell}\right), \qquad \kappa_g = \frac{4\pi^2 A}{\ell^2}\cos\left(\frac{2\pi\xi}{\ell}\right). \tag{10}$$

The rod is assumed to have a continuous flexural rigidity D and we also restrict attention to the case where the surface that the rod is adhering to is flat: $\theta^{*\prime}(\gamma^-) = 0$.

It is convenient to introduce the following notations: $\kappa^+ = \theta^{*\prime}(\gamma^+)$, $(\kappa^+)' = \theta^{*\prime\prime}(\gamma^+)$, and $\kappa_{g_\gamma} = \kappa_g(\gamma^\pm)$. For the example at hand, the boundary condition $(6)_3$ and the function Γ can be expressed in the simplified forms

$$\frac{D}{2}(\kappa^+)^2 = \omega, \tag{11}$$

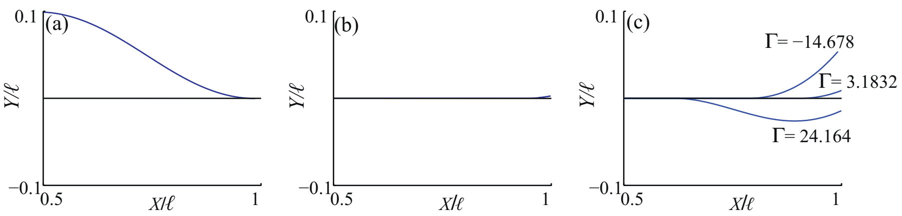

Fig. 2. (a) The natural shape of the elastic rod in the absence of adhesion ($\omega = 0$). (b) Elastic rod with intrinsic curvature adhering to a substrate ($\mathbf{F} = 0\mathbf{E}_1$) along the segment $\xi \in [0, 0.95\,\ell]$. (c) Multiple equilibrium configurations of an elastic rod adhering to an elastic substrate for $\mathbf{F} = -27\mathbf{E}_1$. The solutions where $\Gamma < 0$ are unstable, while the configuration with $\Gamma > 0$ satisfies the necessary condition N1 for stability.

and

$$\Gamma = w(\gamma)(\kappa^+)^2 + D((\kappa^+)' - \kappa'_{g\gamma})\kappa^+ + D\kappa'_{g\gamma}\kappa^+. \quad (12)$$

At equilibrium, $\theta = \theta^*(\xi)$ and $\gamma = \gamma^*$ must satisfy the boundary value problem (4), $(6)_{1,2}$ and (11). The solution $\theta^* = \theta^*(\xi)$ is then used to solve the Riccati equation (7) and, if $w(\xi)$ exists for all $\xi \in [\gamma, \ell]$, then the function Γ can be computed using (12).

We also introduce a range of dimensionless variables and parameters: $\hat{F} = F\ell^2/D$, $\hat{\gamma} = \gamma^*/\ell$, $\hat{\Gamma} = \Gamma\ell^3/D$, $\hat{\omega} = \omega\ell/D$, and $\hat{A} = A/\ell$. We henceforth restrict attention to an example where $\hat{A} = 0.05$ and $\hat{\omega} = 1.7621$. As can be seen from Fig. 3 as F increases from 0, a pair of configurations are possible. Evaluating Γ we find that one of these configurations is unstable and has a jump at $\hat{F} = 16.3$. For reference, $\hat{F} = 2.47$ corresponds to the critical buckling load of a non-adhering cantilever that is clamped at one end and has no intrinsic curvature (i.e. $\kappa_g = 0$). The resulting unstable configuration and the original configuration annihilate each other in a saddle-nod bifurcation at $F \approx 29.8$. For $\hat{F} > 29.8$, a single configuration remains.

An example of the three configurations when $\hat{F} = 27$ is shown in Fig. 2. We suspect that as $\hat{F} \to \approx 38$, the sole configuration also undergoes a bifurcation that is similar to the saddle node bifurcation mentioned earlier. However, we have not been able to numerically find the unstable solution that would feature in this bifurcation. For all of the configurations presented in Fig. 3, the solution to the Riccati equation exists for all $\xi \in [\gamma, \ell]$ and so none of these configurations would be classified as unstable in the classical sense of buckling instability.

It is interesting to note from Fig. 3(a) the presence of hysteresis. As $\hat{F}$ is increased from 0 to $\approx$29.8, the contact length $\ell - \gamma$ gradually decreases. However as $\hat{F}$ is increased beyond 29.8, the contact length suffers a dramatic jump which can be reversed by following the loading path outlined by the arrows in Fig. 3(a).

4. Discussion

Several treatments of adhesion problems (e.g., de Boer and Michalske, 1999; He et al., 2012; Mastrangelo and Hsu, 1993) have featured the computation of the second derivative of Π with respect to γ and stated that the positivity of this derivative is a necessary condition for stability. In line with these treatments, it is interesting to compute the energy of the rod and examine how it changes with the adhesion length γ for a fixed shear load F.

For example, when $\hat{F}$ between 21.7 and 30.0, the governing equations have three solutions, one of which is unstable. Fig. 4 shows a plot of the dimensionless potential energy $\hat{\Pi}$ as a function of the normalized adhesion length $\hat{\gamma}$. The vertical dashed lines correspond to values of $\hat{\gamma}$ at which $\hat{\Pi}$ is locally extremized (see also Fig. 2). In addition to $\Pi = \Pi(\gamma)$, we can also compute the derivatives $\Pi_\gamma = \mathrm{d}\tilde{\Pi}/\mathrm{d}\gamma$ and $\Pi_{\gamma\gamma} = \mathrm{d}^2\tilde{\Pi}/\mathrm{d}\gamma^2$. As expected, the condition $\Pi_\gamma = 0$ is consistent with the natural boundary condition $(6)_3$ and it is also in line with Kendall's seminal work (Kendall, 1971) on problems of this type. However, what was surprising to us were the results that Γ approaches $\Pi_{\gamma\gamma}$ for small deflections (cf. Fig. 3(c)).

Here, we have only examined the case of a non-dimensional intrinsic curvature amplitude $\hat{A} = 0.05$ and adhesion $\hat{\omega} = 1.7621$. As shown in Fig. 3(a), this corresponds to a saddle-node bifurcation at $F = 29.8$, beyond which the contact length suddenly decreases. This critical axial load reduces to $F = 22.1$ for a lower non-dimensional adhesion of $\hat{\omega} = 1$ and $F = 21.7$ when $\hat{\omega}$ remains equal

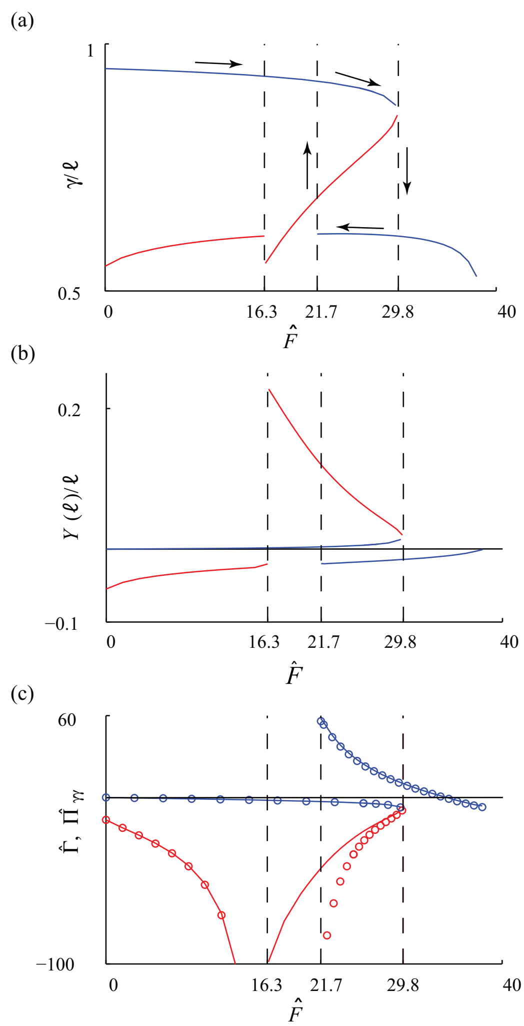

Fig. 3. Plots of (a) adhesion length $\hat{\gamma} = \gamma^*/\ell$ and (b) tip deflection $(Y(\ell) = \mathbf{r}(\ell) \cdot \mathbf{E}_y)/\ell$ as a function of axial load $\hat{F} = F\ell^2/D$ for $\hat{A} = A/\ell = 0.05$. The corresponding values for $\hat{\Gamma} = \Gamma\ell^3/D$ (circles) and $\hat{\Pi}_{\gamma\gamma} = \Pi_{\gamma\gamma}\ell/D$ (curve) are plotted in (c). Blue and red coloring correspond to solutions where $\Gamma > 0$ and unstable solutions where $\Gamma < 0$, respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of the article.)

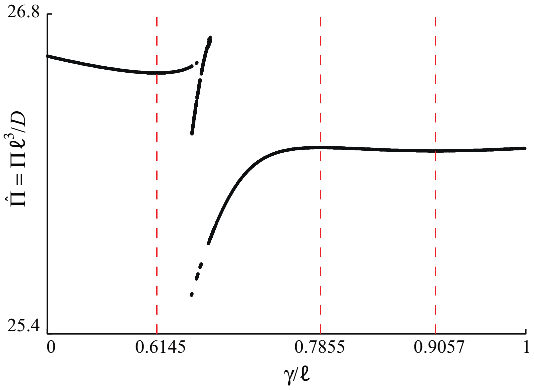

Fig. 4. Plot of potential energy Π as a function of prescribed adhesion length γ for $\mathbf{F} = -27\mathbf{E}_1$. The values of γ for three extrema of Π are distinguished. Two of these extrema are local minima and the third extrema is a local maximum.

to 1.7621 and the intrinsic amplitude is increased to $\hat{A} = 0.1$. Physically, this implies that weaker adhesion and/or greater intrinsic curvature results in a lower resistance to spontaneous detachment under axial load.

In conclusion, we can only state that the treatments of stability featured in de Boer and Michalske (1999), Mastrangelo and Hsu (1993) and He et al. (2012) are compatible with the criterion N1 for the examples discussed in this paper. We emphasize however that a general equivalency proof that $\Pi_{\gamma\gamma} > 0$ if and only if $\Gamma > 0$ is not available at the present time. Nonetheless, the condition $\Gamma > 0$ eliminates the need to numerically compute $\Pi_{\gamma\gamma}$ and enables stability to be examined analytically.

References

Adams, G.G., McGruer, N.E., 2010. A review of adhesion in an ohmic microswitch. Journal of Adhesion Science and Technology 24 (15–16), 2571–2595 http://dx.doi.org/10.1163/016942410X508154

de Boer, M.P., Michalske, T.A., 1999. Accurate method for determining adhesion of cantilever beams. Journal of Applied Physics 86 (2), 817–827 http://dx.doi.org/10.1063/1.370809

Glassmaker, N.J., Hui, C.Y., 2004. Elastica solution for a nanotube formed by self-adhesion of a folded thin film. Journal of Applied Physics 96 (6), 3429–3434 http://dx.doi.org/10.1063/1.1779974

He, L.W., Yan, S.P., Li, B.Q., Chu, J.R., 2012. Directional adhesion behavior of a single elastic fiber. Journal of Applied Physics 112 (1), 013516 http://link.aip.org/link/?JAP/112/013516/1

Hui, C.Y., Jagota, A., Lin, Y.Y., Kramer, E.J., 2002. Constraints on microcontact printing imposed by stamp deformation. Langmuir 18 (4), 1394–1407 http://dx.doi.org/10.1021/la0113567

Jin, M., Bao, Z.B., 2008. Sufficient conditions for stability of Euler elasticas. Mechanics Research Communications 35 (3), 193–200 http://dx.doi.org/10.1016/j.mechrescom.2007.09.001

Kendall, K., 1971. The adhesion and surface energy of elastic solids. Journal of Applied Physics D: Applied Physics 4 (8), 1186–1195 http://dx.doi.org/10.1088/0022-3727/4/8/320

Kuznetsov, V.V., Levyakov, S.V., 2002. Complete solution of the stability problem for elastica of Euler's column. International Journal of Non-Linear Mechanics 37 (6), 1003–1009 http://dx.doi.org/10.1016/S0020-7462(00)00114-1

Maddalena, F., Percivale, D., 2008. Variational models for peeling problems. Interfaces and Free Boundaries 10 (4), 503–5164 http://dx.doi.org/10.4171/IFB/199

Maddocks, J.H., 1984. Stability of nonlinearly elastic rods. Archive for Rational Mechanics and Analysis 85 (4), 311–354 http://dx.doi.org/10.1007/BF00275737

Majidi, C., 2009. Shear adhesion between an elastica and a rigid flat surface. Mechanics Research Communications 36 (3), 369–372 http://dx.doi.org/10.1016/j.mechrescom.2008.10.010

Majidi, C., Adams, G.G., 2010. Adhesion and delamination boundary conditions for elastic plates with arbitrary contact shape. Mechanics Research Communications 37 (2), 214–218 http://dx.doi.org/10.1016/j.mechrescom.2010.01.002

Majidi, C., O'Reilly, O.M., Williams, J.A., 2012. On the stability of a rod adhering to a rigid surface: shear-induced stable adhesion and the instability of peeling. Journal of the Mechanics and Physics of Solids 60 (5), 827–843 http://dx.doi.org/10.1016/j.jmps.2012.01.015

Manning, R.S., 2009. Conjugate points revisited and Neumann-Neumann problems. SIAM Review 51 (1), 193–212 http://dx.doi.org/10.1137/060668547

Manning, R.S., Rogers, K.A., Maddocks, J.H., 1998. Isoperimetric conjugate points with application to the stability of DNA minicircles. Proceedings of the Royal Society A 454 (1980), 3047–3074 http://dx.doi.org/10.1098/rspa.1998.0291

Mastrangelo, C., Hsu, C., 1993. Mechanical stability and adhesion of microstructures under capillary forces. ii. Experiments. Journal of Microelectromechanical Systems 2 (1), 44–55 http://dx.doi.org/10.1109/84.232594

O'Reilly, O.M., 2007. A material momentum balance law for rods. Journal of Elasticity 86 (2), 155–172 http://dx.doi.org/10.1007/s10659-006-9089-6

O'Reilly, O.M., Peters, D.M., 2011. On the stability analyses of three classical buckling problems for the elastic strut. Journal of Elasticity 105 (1–2), 117–136 http://dx.doi.org/10.1007/s10659-010-9299-9

O'Reilly, O.M., Peters, D.M., 2012. Nonlinear stability criteria for tree-like structures composed of branched elastic rods. Proceedings of the Royal Society A 468 (2137), 206–226 http://dx.doi.org/10.1098/rspa.2011.0291

Plaut, R.H., Dalrymple, A.J., Dillard, D.A., 2001. Effect of work of adhesion on contact of an elastica with a flat surface. Journal of Adhesion Science and Technology 15 (5), 565–581 http://dx.doi.org/10.1163/156856101300189934

Plaut, R.H., Williams, N.H., Dillard, D.A., 2001. Elastica analysis of the loop tack adhesion test for pressure sensitive adhesives. Journal of Adhesion 76, 37–53 http://dx.doi.org/10.1080/00218460108029616

Qu, Y., Kim, J., Nguyen, T.D., Lisko, B., Purohit, P.K., McAlpine, M.C., 2011. Enhanced piezoelectricity and stretchability in energy harvesting devices fabricated from buckled pzt ribbons. Nano Letters 11, 1331–1336 http://dx.doi.org/10.1021/nl104412b

Rogers, J.A., Someya, T., Huang, Y., 2010. Materials and mechanics for stretchable electronics. Science 327, 1603–1607 http://dx.doi.org/10.1126/science.1182383

Shih, T.-K., Ho, J.-R., Liao, H.-Y., Chen, C.-F.C.-Y.L., 2008. Fabrication of optical gratings by shrinkage of a rubber material. Thin Film Solids 516 (16), 5339–5343 http://dx.doi.org/10.1016/j.tsf.2007.07.092

Stewart, M., McLachlan, A.D., Calladine, C.R., 1987. A model to account for the elastic element in muscle crossbridges in terms of a bending myosin rod. Proceedings of the Royal Society of London Series B: Biological Sciences 229 (1257), 381–413 http://www.jstor.org/stable/36188

Carmel Majidi[a,*]
Oliver M. O'Reilly[b]
John A. Williams[c]

[a] *Department of Mechanical Engineering, Carnegie Mellon University, Pittsburgh, PA 15213, USA*
[b] *Department of Mechanical Engineering, University of California at Berkeley, Berkeley, CA 94720, USA*
[c] *Department of Engineering, Cambridge University, Cambridge CB2 IPZ, UK*

* Corresponding author. Tel.: +1 4122682492.
E-mail address: cmajidi@andrew.cmu.edu (C. Majidi)
URL: http://www.cmu.edu/me/people/majidi.html (C. Majidi)

4 December 2012

15 January 2013

Available online xxx