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Shear adhesion between an elastica and a rigid flat surface

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ABSTRACT

At the submicron scale, an elastic fiber adheres to a rigid surface when the surface forces induced by electrostatic, capillary, or van der Waals interactions exceed the elastic restoring forces for bending. Adhesion is aided by the application of a shear load to the base of the fiber, which will initiate or increase the length of side contact. The presence of a shear force is necessary for the attachment of a gecko-inspired nanofiber array adhesive, which spontaneously detaches once the shear load is removed. Treating the fiber as an elastica, we derive a relationship between fiber geometry, work of adhesion, applied shear load and length of side contact.

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1. Introduction

Adhesion of elastic bodies is an important issue in engineering structures at the submicron scale, where surface forces generated by electrostatic, chemical, and van der Waals interactions can exceed the restoring forces for elastic deformation. Previously, Majidi (2007) and Majidi et al. (2005) showed that when the work of adhesion induced by these surface effects is sufficiently large, an initially vertical nanofiber will bend over and spontaneously adhere to a horizontal surface. Here, we show that applying a shear load at the base increases the contact length and can also reduce the energy barrier necessary to initiate energetically stable side contact.

In the following analysis, the nanofiber is treated as an elastica and the governing equations are derived from the principle of stationary potential energy. It shows that for certain fiber geometries, side contact is only possible when a sufficiently large shear load is applied to the base of the fiber. This result may explain the shear-activated property observed with the gecko-inspired adhesive introduced by Lee et al. (2008). As with the natural gecko, the synthetic adhesive is an array of vertically aligned, high aspect ratio nanofibers (length $L = 20 \mu\text{m}$, radius $R = 300 \text{ nm}$) composed entirely of a stiff material (polypropylene, elastic modulus $E = 1 \text{ GPa}$) that only adheres under shear loading – once the shear load is removed the adhesive spontaneously delaminates from the surface.

2. Preliminaries

The fiber is treated as an elastica Ω parameterized by the convected coordinate ξ . To distinguish between the contacting and non-contacting portions of the elastica, Ω is decomposed into closed complementary subregions Ω_α and Ω_β , which are identified with the coordinates $[0, a]$ and $[a, L]$, respectively, where $\xi = 0$ is the base of the elastica, L is the rod length, and a is the position of the “crack tip” formed between the elastica and contacting surface. The elastica and coordinate system are illustrated in Fig. 1.

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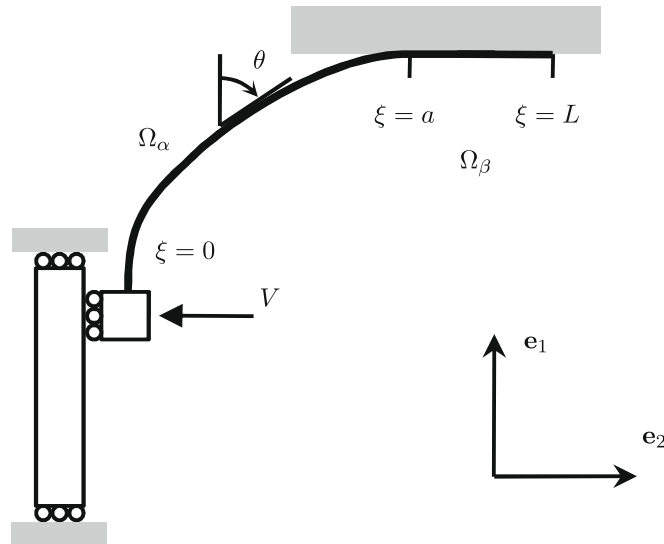


Fig. 1. Illustration of the elastica in side contact under a shear load V .

In its natural (undeformed) configuration, the elastica is vertical and points along Ω have the position $\mathbf{R} = \xi \mathbf{e}_1$, where $\{\mathbf{e}_1, \mathbf{e}_2\}$ is the fixed Cartesian frame. During deformation, the elastica bends with a slope $\theta = \theta(\xi)$ with respect to the \mathbf{e}_1 axis. In this configuration, the position of the material points are defined by the mapping $\mathbf{r} = \mathbf{r}(\xi) = x\mathbf{e}_1 + y\mathbf{e}_2$, which is related to the slope through the identity $\mathbf{r}' = \cos \theta \mathbf{e}_1 + \sin \theta \mathbf{e}_2$. Here, the prime denotes the derivative with respect to ξ . The function $\theta = \theta(\xi)$ is restricted by the boundary conditions $\theta(0) = 0$ and $\theta(\xi) = \pi/2 \forall \xi \in [a, L]$.

Under pure shear loading, the elastica is subject to a force $\mathbf{F} = -V\mathbf{e}_2$ at $\xi = 0$. Define ω as the mechanical work required to overcome adhesion and displace a unit length of the fiber from a position of stable contact to a distance of infinite separation between the fiber and substrate. For the surface forces of interest, most of this work is performed over only a few nanometers of displacement and so it will be assumed that adhesion energy is absent along the non-contacting portion Ω_α . The total potential energy of the system may thus be expressed as

$$\begin{aligned} \Phi &= \int_0^a \frac{1}{2} EI(\theta')^2 d\xi - \int_a^L \omega d\xi + \mathbf{F} \cdot (\mathbf{r}(L) - \mathbf{r}(0)), \\ &= \int_0^a \left\{ \frac{1}{2} EI(\theta')^2 - V \sin \theta \right\} d\xi - (V + \omega)(L - a), \end{aligned} \tag{1}$$

where E is the elastic modulus and I is the area moment of inertia for the elastica cross section.

3. Analysis

At equilibrium, the potential energy Φ is stationary with respect to infinitesimal, kinematically admissible variations in θ and a . The stationary condition for variations in θ simply leads to the Euler–Lagrange differential equation

$$\theta'' + \frac{V}{EI} \cos \theta = 0 \quad \forall \xi \in [0, a], \tag{2}$$

which is solved for the boundary conditions $\theta(0) = 0$ and $\theta(a) = \pi/2$. The condition that Φ is stationary with respect to a implies $d\Phi/da = 0$, which yields the natural boundary condition (Bottega, 1991; Oyharcabal and Frisch, 2005; Seifert, 1991)

$$\omega = \frac{1}{2} EI\{\theta'(a)\}^2. \tag{3}$$

In summary, the unknowns $\theta = \theta(\xi)$ and a are the solutions to the boundary value problem

$$\theta'' + \frac{V}{EI} \cos \theta = 0 \quad \theta(0) = 0 \quad \theta(a) = \frac{\pi}{2} \quad \theta'(a) = \sqrt{\frac{2\omega}{EI}}. \tag{4}$$

4. Solution

Eq. (4)₁ has the general solution

$$\theta(\xi) = 2 \operatorname{am} \left(\frac{\xi + C_2}{2} \sqrt{2\kappa^2 + C_1}, \frac{4\kappa^2}{2\kappa^2 + C_1} \right) - \frac{\pi}{2}, \tag{5}$$

where $\text{am}(u, m)$ is the Jacobi amplitude, m is the modulus, $\kappa = \sqrt{V/EI}$, and C_1 and C_2 are the constants of integration. Next, the boundary condition $\theta(0) = 0$ implies

$$\text{am}\left(\frac{C_2}{2} \sqrt{2\kappa^2 + C_1}, \frac{4\kappa^2}{2\kappa^2 + C_1}\right) = \frac{\pi}{4}. \tag{6}$$

The inverse function of am is given by the elliptic integral of the first kind: $\text{am}^{-1}(\phi, m) = u = F(\phi, m)$ (Byrd and Friedman, 1971). Defining $C = \sqrt{4\kappa^2/(2\kappa^2 + C_1)}$ it follows from (6) that $C_2 = (C/\kappa)F(\pi/4, C^2)$. Thus

$$\theta = 2 \text{am}\left(\frac{\kappa\xi}{C} + F\left(\frac{\pi}{4}, C^2\right), C^2\right) - \frac{\pi}{2}. \tag{7}$$

Next, consider the boundary condition $\theta(a) = \pi/2$. Substituting this into (7) and solving for a yields

$$a = \frac{C}{\kappa} \left\{ K(C^2) - F\left(\frac{\pi}{4}, C^2\right) \right\} \tag{8}$$

where $K(m) = F(\pi/2, m)$ is the complete elliptic integral of the first kind. Substituting the expressions for θ and a into the natural boundary condition (4)₄ implies

$$\frac{2\kappa}{C} \text{dn}(K(C^2), C^2) = \sqrt{\frac{2\omega}{EI}}. \tag{9}$$

The elliptic function $\text{dn}(u, m)$ is defined as $\text{dn}(u, m) = \sqrt{1 - m \sin^2 \phi}$. For $K(m)$, $\phi = \pi/2$ and so $\text{dn}(K(m), m)$ reduces to $\sqrt{1 - m}$. Hence, (9) implies $C = (1 + \omega/2V)^{-1/2}$. Lastly, substituting C and κ into the expressions for (7) and (8) yield

$$\theta(\xi) = 2 \text{am}\left(\xi \sqrt{\frac{V + \omega/2}{EI}} + F\left(\frac{\pi}{4}, \frac{1}{1 + \omega/2V}\right), \frac{1}{1 + \omega/2V}\right) - \frac{\pi}{2} \tag{10}$$

and

$$a = \left\{ K\left(\frac{1}{1 + \omega/2V}\right) - F\left(\frac{\pi}{4}, \frac{1}{1 + \omega/2V}\right) \right\} \sqrt{\frac{EI}{V + \omega/2}}. \tag{11}$$

5. Discussion

The solutions (10) and (11) indicate the elastica configuration for a prescribed shear load V , work of adhesion per unit length of side contact ω , elastic modulus E , and area moment of inertia I . Define the characteristic length $\ell = \sqrt{EI/V}$ and the non-dimensional parameters $\hat{a} = a/\ell$, $\hat{\xi} = \xi/\ell$, and $\alpha = 1/(1 + \omega/2V)$. It then follows that:

$$\theta(\hat{\xi}) = 2 \text{am}\left(\frac{\hat{\xi}}{\sqrt{\alpha}} + F\left(\frac{\pi}{4}, \alpha\right), \alpha\right) - \frac{\pi}{2} \quad \text{and} \quad \hat{a} = \left\{ K(\alpha) - F\left(\frac{\pi}{4}, \alpha\right) \right\} \sqrt{\alpha}. \tag{12}$$

Since both ω and V are non-negative, $\alpha \in [0, 1]$.

A plot of \hat{a} versus α is presented in Fig. 2a. For a “sticky” fiber, where ω is large, and $\alpha \rightarrow 0$, \hat{a} vanishes. Physically this means that the length of the non-contacting portion Ω_α goes to zero and the entire fiber will spontaneously bend over

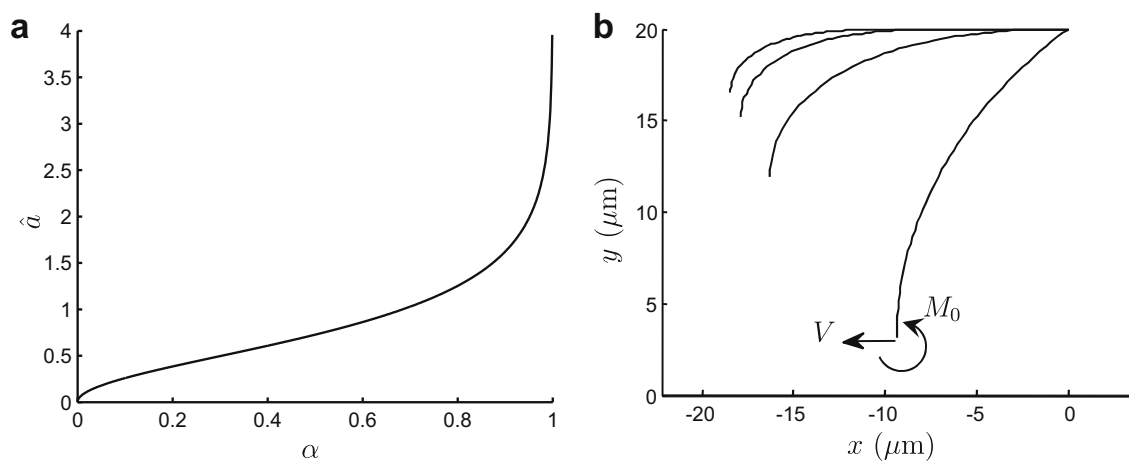


Fig. 2. (a) Plot of $\hat{a} = a\sqrt{V/EI}$ versus $\alpha = 1/(1 + \omega/2V)$ and (b) shape of elastica for, from right, $V = 10, 160, 500, 970$ nN; $E = 1$ GPa, $L = 20$ μm , $I = 0.0064$ μm^4 , $\omega = 1.7$ nN; $M_0 = EI\theta'$ represents the moment acting on the base of the elastica.

and adhere to the surface along its side. In contrast, for a non-sticky fiber ($\omega \rightarrow 0$ and $\alpha \rightarrow 1$), the plot shows that $\hat{a} \rightarrow \infty$. Since side contact only occurs for fibers of length $L < \hat{a}\ell$, this implies that a non-sticky fiber will not adhere in side contact.

Consider the nanofibers contained in the synthetic gecko adhesive presented in Lee et al. (2008). The fibers have an elastic modulus $E = 1$ GPa, length $L = 20$ μm , radius $R = 0.3$ μm , and area moment of inertia $I = 0.0064$ μm^4 . Majidi et al. (2005) show that for an elastic cylinder in contact with a flat surface

$$\omega = 6 \left\{ \frac{(1 - \nu^2)R^2 W_{\text{ad}}^4}{\pi E} \right\}^{1/3}, \quad (13)$$

where ν is Poisson's ratio and W_{ad} is the work of adhesion per unit area of contact. For polypropylene, $\nu = 0.3$ and $W_{\text{ad}} = 30$ mJ/m^2 , and so calculating (13) yields $\omega = 1.7$ nN . Also, following a Kendall peel model of adhesion, Lee et al. (2008) show that V is bounded above by $V^* = \sqrt{2E\pi R^2 \omega} = 970$ nN . These values correspond to $\ell = 2.6$ μm and $\alpha = 0.999$. Substituting α into (12)₂, $\hat{a} = 4$, and so $a = \hat{a}\ell = 10.4$ μm . Since this is less than $L = 20$ μm , the fiber will engage in side contact over a length of 9.6 μm . This is close to the value calculated by Lee et al. (2008), who obtain a numerical solution to (4) using a finite difference program. Next, θ is evaluated for $\xi \in [0, \hat{a}]$ by substituting α into (12)₁. Integrating $\sin(\theta)$ and $\cos(\theta)$ yields the coordinates of the material points along Ω_α which are plotted in Fig. 2b for $V \leq V^*$. Side contact occurs for $V > V_{\text{cr}}$, where V_{cr} is the critical value at which $L = \hat{a}\ell$ and is computed as 160 nN . That is, side contact will only be possible if the interface can support a shear load of greater than 160 nN .

In closing, elastica theory and stationary principles are used to determine the adhesion of an elastic fiber to a rigid surface. If the work of adhesion generated by surface forces is sufficiently large the fiber will spontaneously adhere to the surface and make side contact. For the nanofibers calculated in the synthetic gecko adhesive presented in Lee et al. (2008), side contact is only possible when the base of the fiber is subject to a shear load. This feature allows for a controllable adhesive that only attaches to a surface under shear loading and spontaneously detaches once the shear load is removed.

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