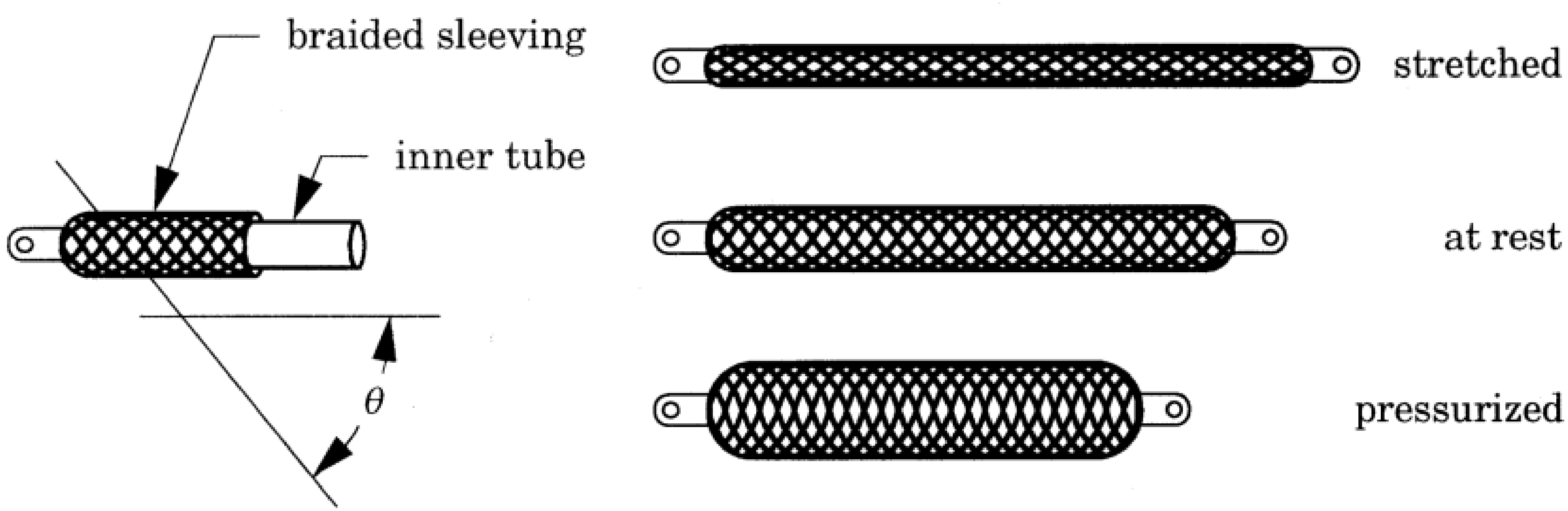
McKibben Actuators

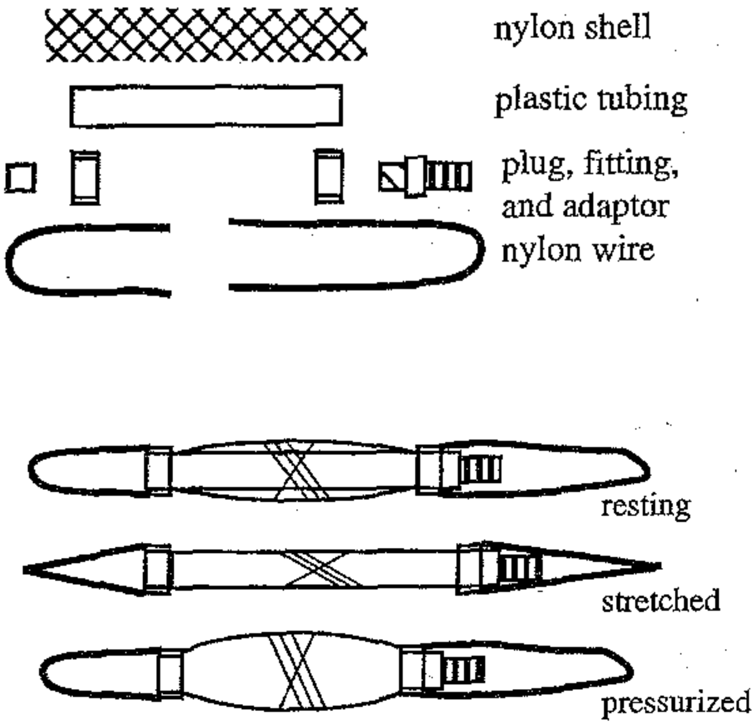


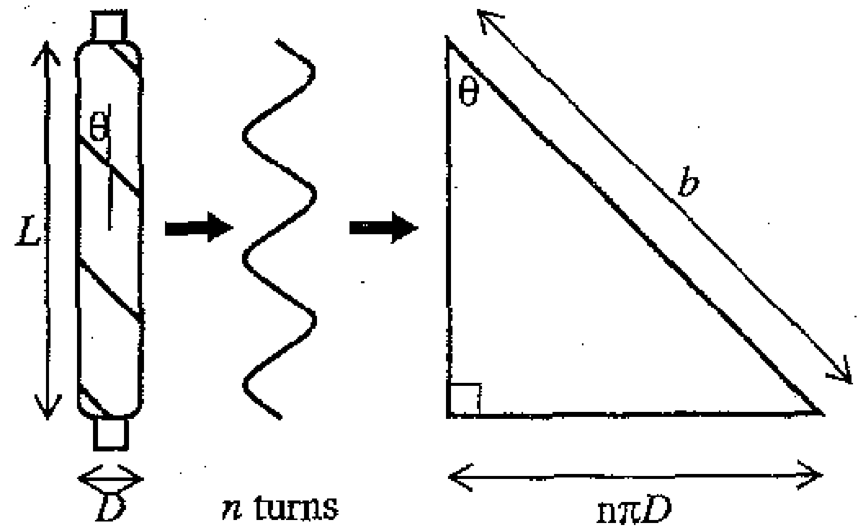
Daerden & Lefeber, *Euro. J. Mech. Environ. Eng.* 2002.

Naturally Flexible & Elastic
Tunable Elastic Stiffness
Lightweight

McKibben Actuators – Theory

CHOU AND HANNAFORD: MEASUREMENT AND MODELING OF ARTIFICIAL MUSCLES
IEEE TRANSACTIONS ON ROBOTICS AND AUTOMATION, VOL. 12, NO. 1, FEBRUARY 1996





$$L = b\cos\theta$$

$$D = \frac{b\sin\theta}{n\pi}$$

Cylinder:

$$\lambda_1 = L/L_0$$
$$\lambda_2 = R/R_0$$
$$\lambda_3 = 1/\lambda_1\lambda_2 = L_0R_0/LR$$

$$\Pi = WV_0 - pV_e - FL$$

$$V_0 = 2\pi R_0 t_0 L_0 \qquad V_e = \pi R^2 L$$

$$W = \frac{E}{24}\left(\lambda_1^4 + \lambda_2^4 + \lambda_3^4 - 3\right)$$

$$\Pi = \frac{1}{12}\pi E R_0 t_0 L_0 \left(\lambda_1^4 + \lambda_2^4 + \frac{1}{\lambda_1^4\lambda_2^4} - 3\right) - \pi R^2 L p - FL$$

$$R = \frac{D}{2} = \frac{b\sin\theta}{2n\pi} = \frac{b}{2n\pi}\sqrt{1-\cos^2\theta}$$

$$L = b\cos\theta$$

$$\therefore R = \frac{b}{2n\pi}\sqrt{1-\frac{L^2}{b^2}}$$

Prescribed internal pressure p and external tensile load F. Solve for L: $\Pi = \Pi\left(L; p, F\right)$ $\qquad \dfrac{d\Pi}{dL} = 0$

$$\frac{d\Pi}{dL} = 0 \Rightarrow F = V_0 \frac{dW}{dL} - p\frac{dV_e}{dL}$$

$$= \frac{d}{dL}\left\{\frac{1}{12}\pi E R_0 t_0 L_0 \left(\lambda_1^4 + \lambda_2^4 + \frac{1}{\lambda_1^4 \lambda_2^4} - 3\right) - \pi R^2 L p\right\}$$

$$R := \frac{b}{2\,n\,\pi}\sqrt{1 - \frac{L^2}{b^2}};$$

$$\lambda 1 := \frac{L}{L0};$$

$$\lambda 2 := \frac{R}{R0};$$

$$\lambda 3 := \frac{1}{\lambda 1\, \lambda 2};$$

$$W := \frac{Y}{24}\left(\lambda 1^2 + \lambda 2^2 + \lambda 3^2 - 3\right);$$

$$V0 := 2\,\pi\, R0\; t0\; L0;$$

$$Ve := \pi\, R^2\, L;$$

$$\partial_L\,(W\,V0 - p\,Ve)$$

Pressure Controlled

Elasticity Controlled

$$\boxed{\frac{L^2\,p}{2\,n^2\,\pi} - \frac{b^2\left(1 - \frac{L^2}{b^2}\right)p}{4\,n^2\,\pi}} + \boxed{\frac{1}{12}\,L0\,\pi\,R0\left(\frac{2\,L}{L0^2} - \frac{L}{2\,n^2\,\pi^2\,R0^2} + \frac{8\,L0^2\,n^2\,\pi^2\,R0^2}{b^4\,L\left(1 - \frac{L^2}{b^2}\right)^2} - \frac{8\,L0^2\,n^2\,\pi^2\,R0^2}{b^2\,L^3\left(1 - \frac{L^2}{b^2}\right)}\right) t0\,Y}$$

To solve for L, we must perform a numerical root finding.
First, we must get an expression for dΠ/dL:

Maple Code:

$$R := \frac{b}{2\,n\,\pi}\sqrt{1-\frac{L^2}{b^2}}:$$

$$\lambda_1 := \frac{L}{L0}:$$

$$\lambda_2 := \frac{R}{R0}:$$

$$\lambda_3 := \frac{1}{\lambda_1 \lambda_2}:$$

$$W := \frac{E}{24}\left(\lambda_1^4+\lambda_2^4+\lambda_3^4-3\right):$$

$$V0 := 2\,\pi\,R0\,t0\,L0:$$

$$Ve := \pi\,R^2\,L:$$

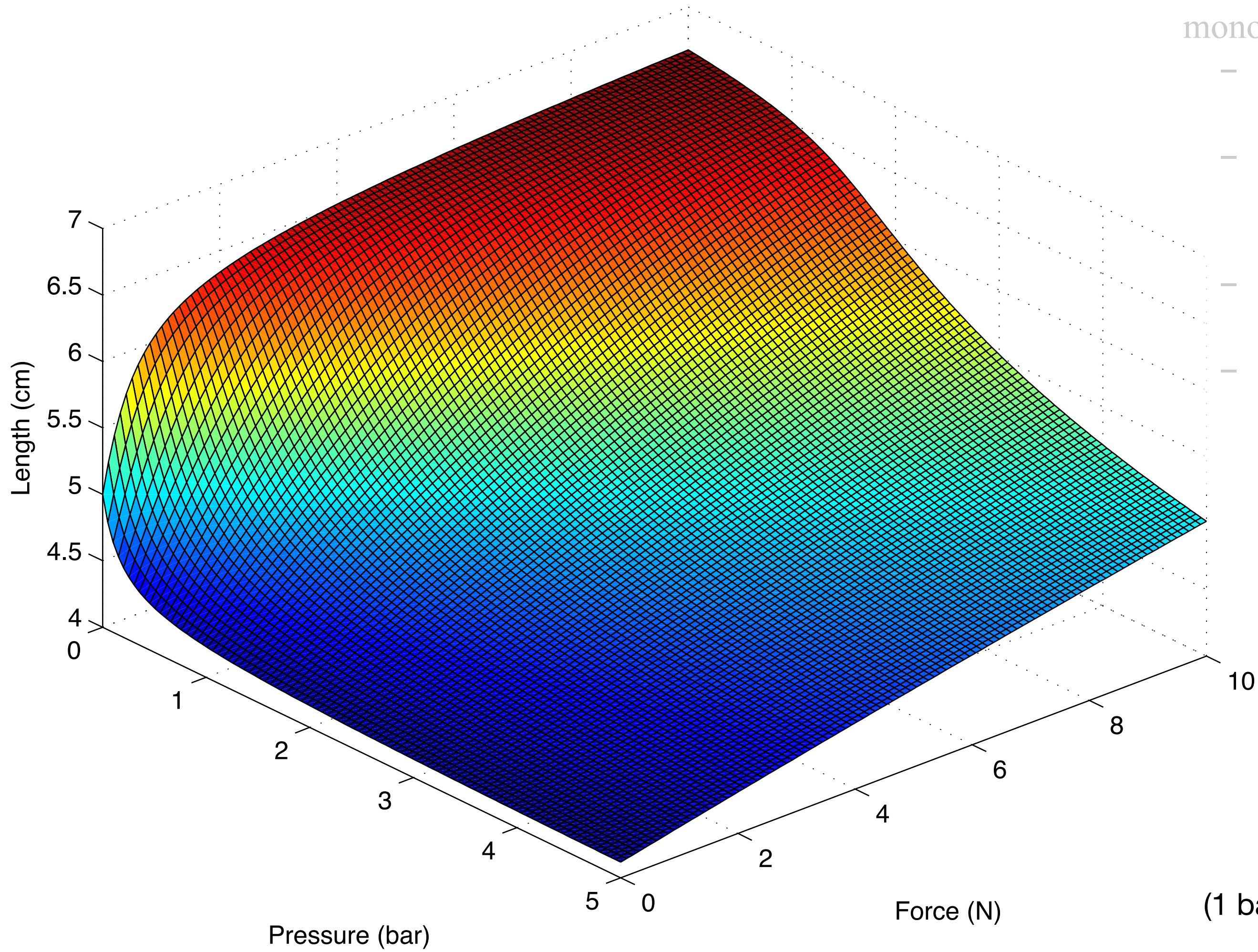
$$\Pi := W\,V0 - p\,Ve - F\,L:$$

$$\frac{\mathrm{d}}{\mathrm{d}L}\Pi$$

$$\frac{1}{12}E\left(\frac{4\,L^3}{L0^4}-\frac{1}{4}\,\frac{b^2\left(1-\frac{L^2}{b^2}\right)L}{n^4\,\pi^4\,R0^4}-\frac{64\,L0^4\,n^4\,\pi^4\,R0^4}{L^5\,b^4\left(1-\frac{L^2}{b^2}\right)^2}+\frac{64\,L0^4\,n^4\,\pi^4\,R0^4}{L^3\,b^6\left(1-\frac{L^2}{b^2}\right)^3}\right)\pi\,R0\,t0\,L0+\frac{1}{2}\,\frac{p\,L^2}{\pi\,n^2}-\frac{1}{4}\,\frac{p\,b^2\left(1-\frac{L^2}{b^2}\right)}{\pi\,n^2}-F$$

Theoretical Results

R_0 = 2.5 mm E = 1 MPa
L_0 = 50 mm $\theta_0 = \pi/4$
t_0 = 0.1 mm $b = L_0/\cos(\theta_0)$ = 70.7 mm
n = 3.18



(1 bar = 100 kPa)

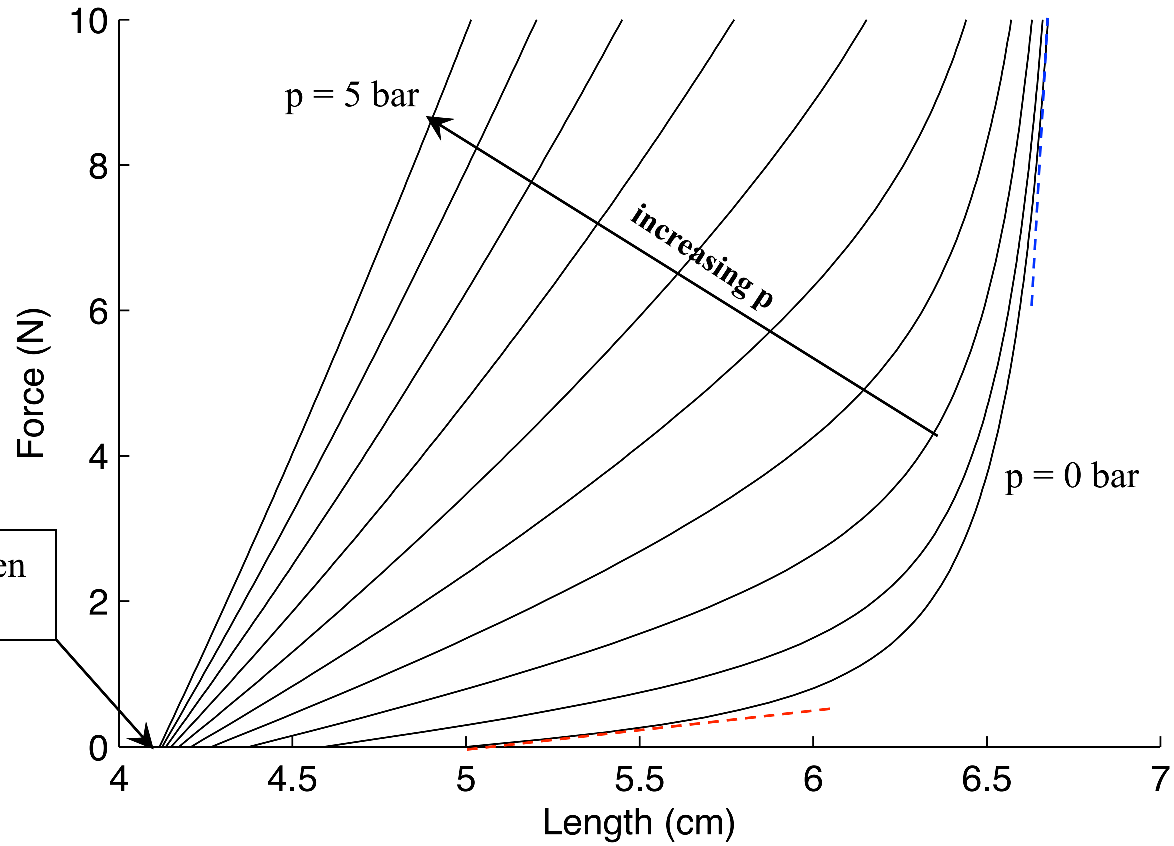
Observations

- When F = 0 and p > 100 kPa, the "rest" length reduces to ≈4.08 cm
- Tensile stiffness does not increase monotonically with force
  - Highly non-linear for moderate pressures
  - When p = 0, high compliance is followed by almost infinite stiffness
  - Infinite stiffness corresponds to $\theta \approx 0$, $L \approx b$ = 7.07 cm
  - Linear for high pressures; linearized stiffness increases with increasing p

Theoretical Results

$$\frac{L^2 p}{2 n^2 \pi} - \frac{b^2 \left(1 - \frac{L^2}{b^2}\right) p}{4 n^2 \pi} + \frac{1}{12} L0\, \pi\, R0 \left(\frac{2 L}{L0^2} - \frac{L}{2 n^2 \pi^2 R0^2} + \frac{8 L0^2 n^2 \pi^2 R0^2}{b^4 L \left(1 - \frac{L^2}{b^2}\right)^2} - \frac{8 L0^2 n^2 \pi^2 R0^2}{b^2 L^3 \left(1 - \frac{L^2}{b^2}\right)} \right) t0\, Y$$

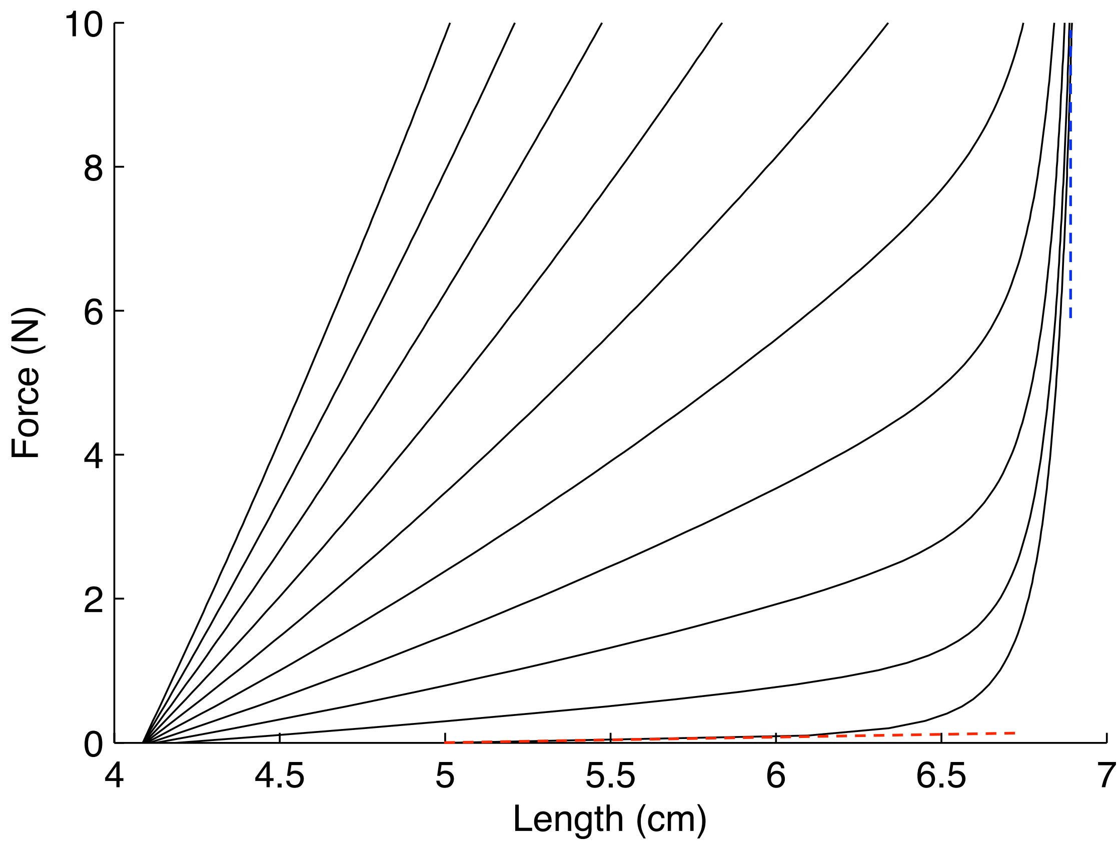
$R_0 = 2.5$ mm $E = 1$ MPa
$L_0 = 50$ mm $\theta_0 = \pi/4$
$t_0 = 0.1$ mm $b = L_0/\cos(\theta_0) = 70.7$ mm
$n = 3.18$



For large p, rest length (i.e. L when F = 0) approaches L* = 40.8 mm

Theoretical Results

R_0 = 2.5 mm   E = 1 MPa
L_0 = 50 mm $\theta_0 = \pi/4$
t_0 = 0.01 mm $b = L_0/\cos(\theta_0)$ = 70.7 mm
n = 3.18



Observations

As t0 → 0, the elasticity of the shell wall can be ignored.

- The actuator contracts to a rest length L* = 40.8 mm for *any* pressure $p > 0$
- When $p = 0$, only a little amount of tension is required to straighten out the helical fibers ($\theta \approx 0$, $L \approx b$ = 7.07 cm)
- For large pressures, the solution is similar to before (i.e. shell wall thickness only matters when p is small)

Suppose we ignore the elasticity of the cylinder wall.

$$\Pi \approx -pV_e - FL$$

$$\Rightarrow F \approx -p\frac{dV_e}{dL} = -p\frac{d}{dL}\left\{\pi\left(\frac{b}{2n\pi}\right)^2\left(1-\frac{L^2}{b^2}\right)L\right\}$$

$$\therefore F \approx \frac{pb^2}{4\pi n^2}\left\{\frac{3L^2}{b^2}-1\right\}$$

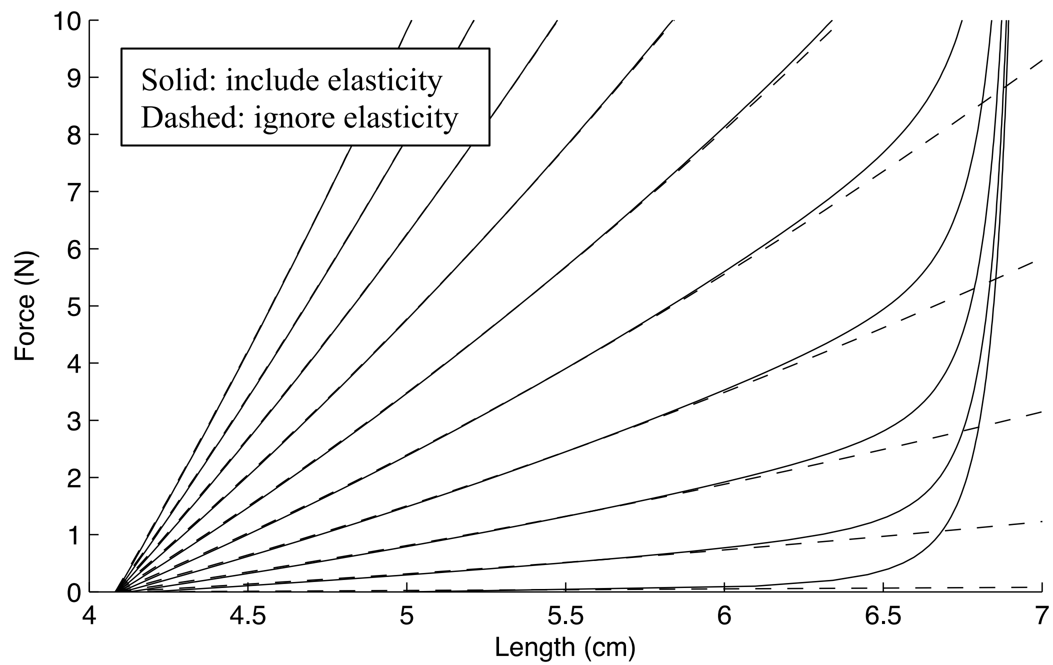
$$\Rightarrow L \approx \sqrt{\frac{4n^2\pi F}{3p}+\frac{b^2}{3}}$$

For F = 0 and p > 0,

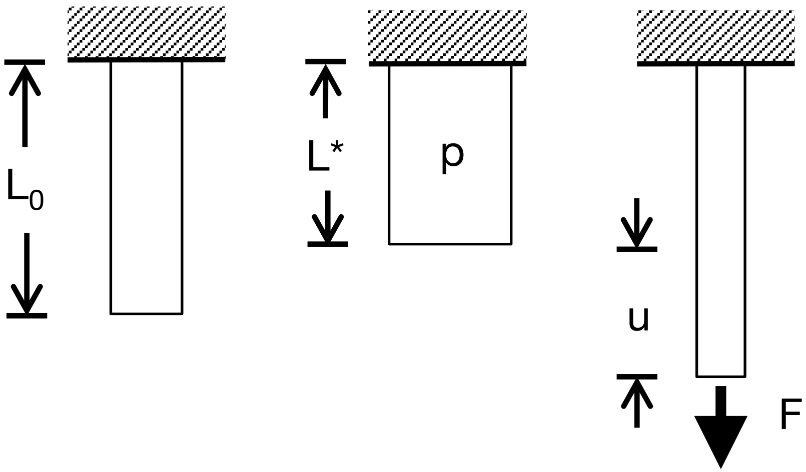
$$L = L^* = \frac{b}{\sqrt{3}}$$

$R_0 = 2.5$ mm
$L_0 = 50$ mm
$t_0 = 0.01$ mm

$E = 1$ MPa
$\theta_0 = \pi/4$
b = 7.07 mm
L* = 4.08 cm ✓

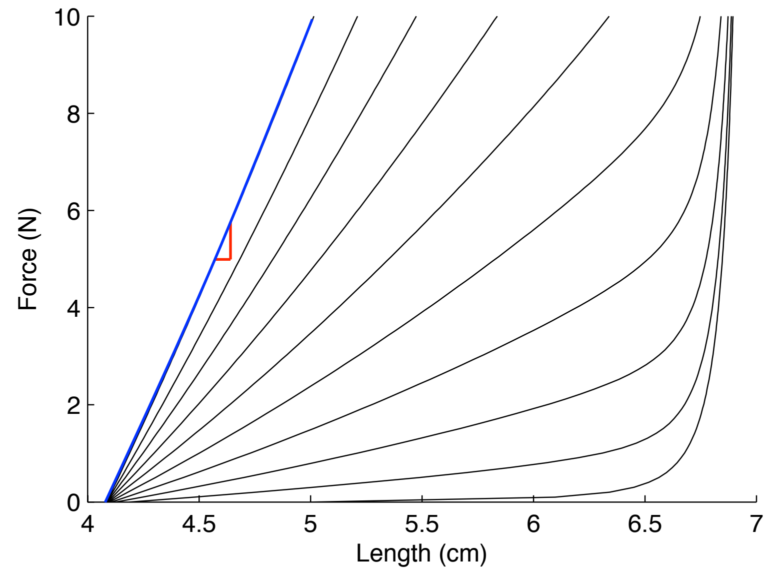


McKibben Actuators – Effective Spring Constant



$$F = \frac{pb^2}{4\pi n^2}\left\{\frac{3}{b^2}\left(\frac{b}{\sqrt{3}} + u\right)^2 - 1\right\} \approx ku$$

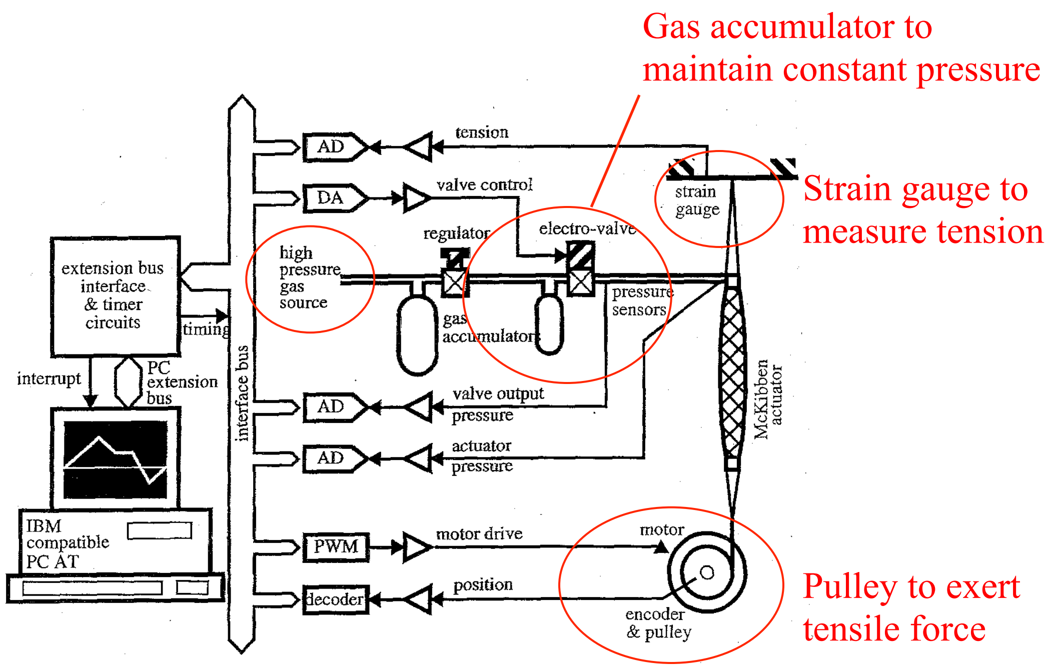
$$k = \left.\frac{dF}{du}\right|_{u=0} = \left\{\frac{b\sqrt{3}}{2\pi n^2}\right\} p$$

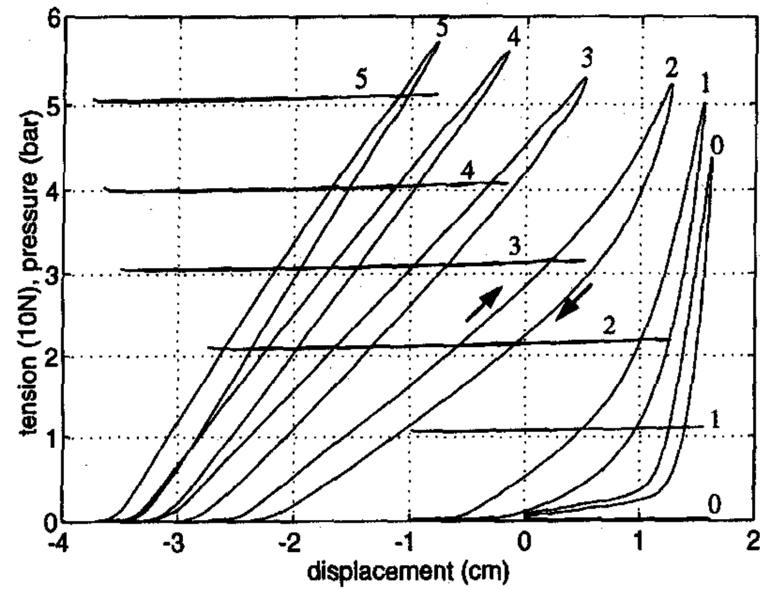


b = 7.07 cm p = 0.5 MPa

n = 3.18 **k = 9.6 N/cm ✓**

Experiment





Nylon Shell
$L_0 = 14$ cm
$R = 5.5$ mm (@ 5 bar)

- Rest length changes with pressure; elasticity of the cylinder wall matters!
- Hysteresis – why?

$L^* \approx (14 - 3.5)\text{cm} = 10.5$ cm (estimated)
$b = 1.732L^* = 18.2$ cm

$$n = \frac{b}{2R\pi}\sqrt{1 - \frac{L^2}{b^2}} = 4.3$$

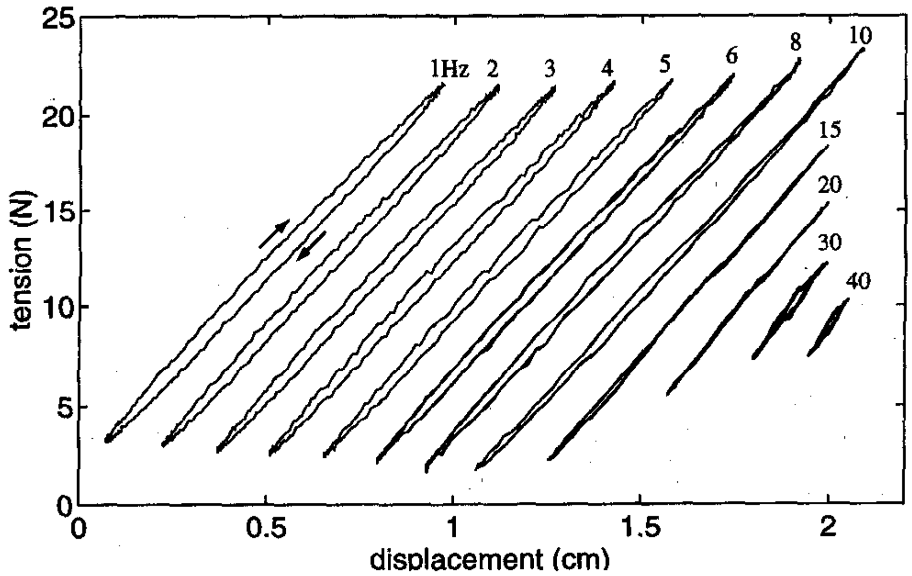
$$k = \left\{\frac{b\sqrt{3}}{2\pi n^2}\right\} p = 13.5\frac{\text{N}}{\text{cm}}$$ (exp: ≈20 N/cm)

Hysteresis

Two potential sources:

Viscous Damping
- viscoelasticity of shell wall
- viscous drag of pressurized air
- External Air drag
- increase with velocity/frequency

Coulombic Friction
- Sliding friction between fibers and shell wall
- invariant to loading frequency



Little change in the width of the tension-displacement loop (except at high frequencies) ⇒ Hysteresis primarily governed by friction

Efficiency

Quasi-static (no KE, Isothermal), Constant Pressure Loading

Hypothetical piston-valve system for determining energy required to pressurize air. Source tubing is assumed to have zero/negligible volume.

Step 1 (D → A)

- Actuator starts out with ambient pressure (P_0 = 1 bar) and rest length L_0 = 14 cm.
- Actuator is stretched to a length $L_2 \approx$ 15.5 cm.

Step 2 (A → B)

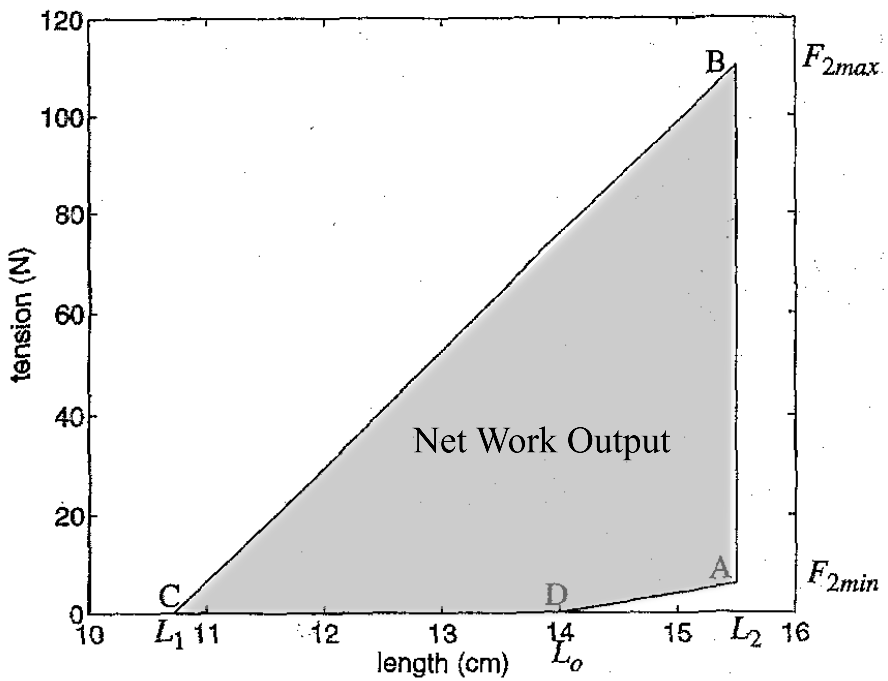
- Internal pressure is increased from P_0 = 1 bar to P_h = 6 bar.
- The length L = L_2 is held fixed by increasing the tension from $F_{2min} \approx 5$ to $F_{2max} \approx 110$ N .

Step 3 (B → C)

- The pressure is kept fixed at 6 bar while the tension is released.
- The length contracts to $L_1 \approx$ 10.7 cm.

Step 4 (C → D)

- Release pressure: P → P_0
- Actuator lengthens back to L_0 = 14 cm.



$$W_a = \frac{1}{2}[(L_2 - L_1)F_{2\,\max} - (L_2 - L_0)F_{2\,\min}]$$

Efficiency

V = total volume of air in compressor and actuator
$V_i = 55\ cm^3$ = initial total air volume (after Step 1).
P = air pressure in compressor and actuator

Step 2: A → B

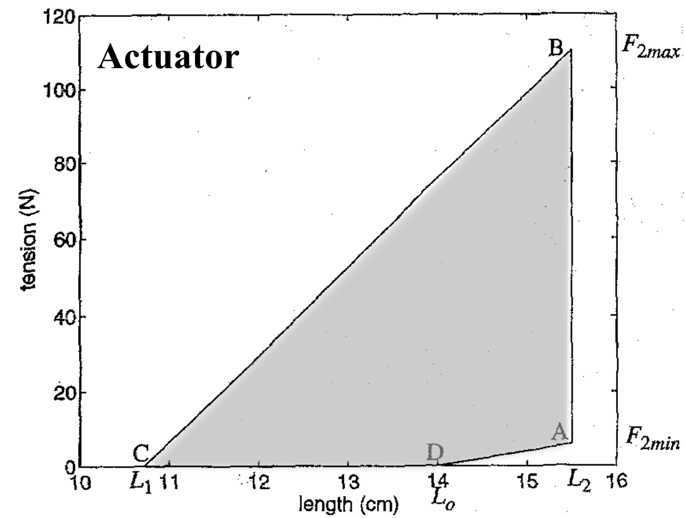
- Air pressure is increased (from $P_0 = 1$ bar to $P_h = 6$ bar) by reducing volume of compressor (by 45 cm^3).
- Tension is applied to actuator to maintain constant length ($L_2 \approx 15.5$ cm) and actuator
- Total final volume is $V_1 = 10\ cm^3$.
- Since air doesn't enter or leave the system, PV = constant, i.e. $P_0V_i = P_hV_1$

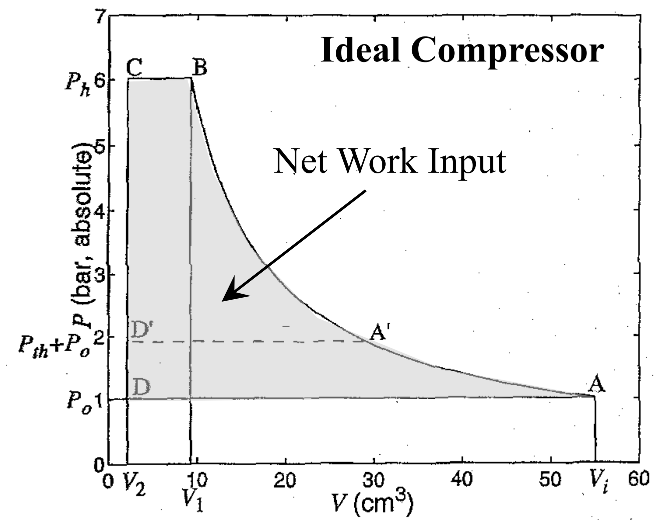
Step 3: B → C

- The pressure is kept fixed at 6 bar while the tension is released.
- Actuator contracts to $L_1 \approx 10.7$ cm.
- Compressor fully collapses, i.e. all the air is in the actuator ($V = V_2 \approx 2.5\ cm^3$)

Step 4: C → D

- Release pressure (exhale) to environment
- Actuator lengthens back to $L_0 = 14$ cm.
- Volume remains constant (i.e. equal to volume V_2 in actuator)





Efficiency

$$W_{01} = -\int_{V_i}^{V_1} (P - P_0)dV = -\int_{V_i}^{V_1} PdV - P_0(V_i - V_1)$$

$$= -P_h V_1 \int_{V_2}^{V_1} \frac{dV}{V} - P_0 V_i \left(1 - \frac{P_0}{P_h}\right)$$

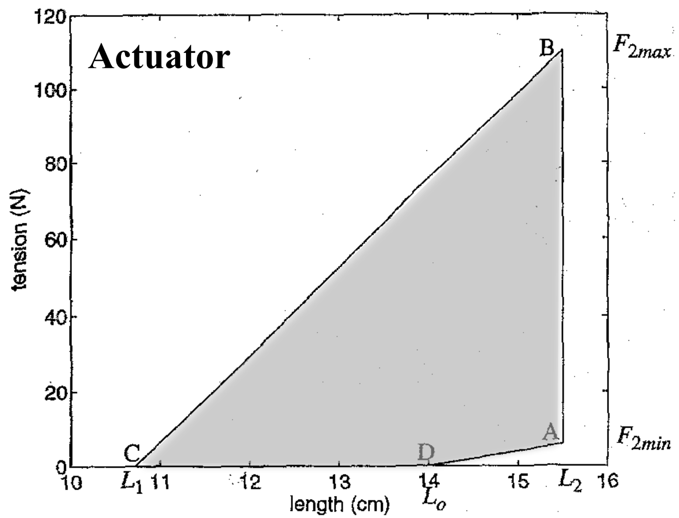
$$= P_h V_1 \left(\log \frac{P_h}{P_0} - 1 + \frac{P_0}{P_h} \right).$$

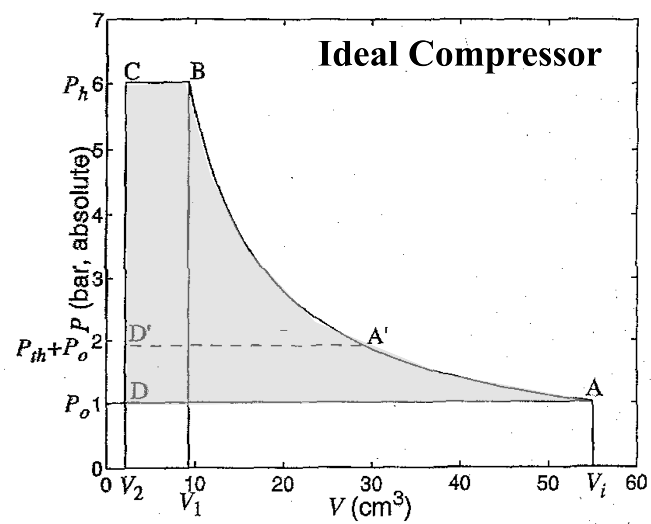
$$W_{12} = (P_h - P_0)(V_1 - V_2)$$

$$E_a = \frac{W_a}{W_{01} + W_{12}}$$

Measured: $E_a \approx 0.3$

- Energy lost when exhaling pressurized air to environment
- Actuator not performing mechanical work in Step 2
  - work input from compressor (W_{01}) is all wasted in Step 4
  - Step 2 is necessary for actuator to perform work in step 3
- Improve efficiency by exhaling/inhaling with a gas accumulator: $E_a \rightarrow$ **0.5**!





McKibben Actuators

The Mechanical Properties of the McKibben Actuators and Biological Muscles

properties	units	McKibben actuators @ 5 bar			Biological muscles
		Nylon shell	Fiberglass shell	Bridgestone	
Resting length (L_o)	cm	14.0	20.0	14.7	
Dynamic range	L_o	0.75 - 1.1	0.86-1.14	0.79 - 1.02	0.64 - 1.12
Maximum tension	N	110 @ 1.1 L_o	56 @ 1.15 L_o	260 @ 1.02 L_o	
Stiffness	N/L_o	314	200	1130	
Work per cycle	NL_o	19	7.8	30	
Cross section area	cm^2	0.95 @ 0.75 L_o	0.64 @ 0.85 L_o	2.0 @ 0.78 L_o	
Tension intensity	N/cm^2	116	88	130	35
Stiffness intensity	$N/L_o cm^2$	331	313	565	73
Work density per cycle	J/cm^3	0.20	0.12	0.15	0.13
Coulomb friction	N	2.5	5	5	0
Maximum velocity	L_o/s	> 6.19			2 - 8 or 20
Average power density	W/cm^3	1.1			
Peak power density	W/cm^3	2.65			0.70
Energy efficiency	-	0.32 - 0.49			0.2 - 0.25

Klute & Hannaford (1998)

Solution w/ Mooney-Rivlin

ref. Glenn Klute & Blake Hannaford

"Accounting for Elastic Energy Storage in McKibben Artificial Muscle Actuators" Journal of Dynamic Systems, Measurement, and Control, vol. 122, pg. 386-388 (1998).

$$F_{mr} = V_0 \frac{dW}{dL} - p\frac{dV_e}{dL}$$

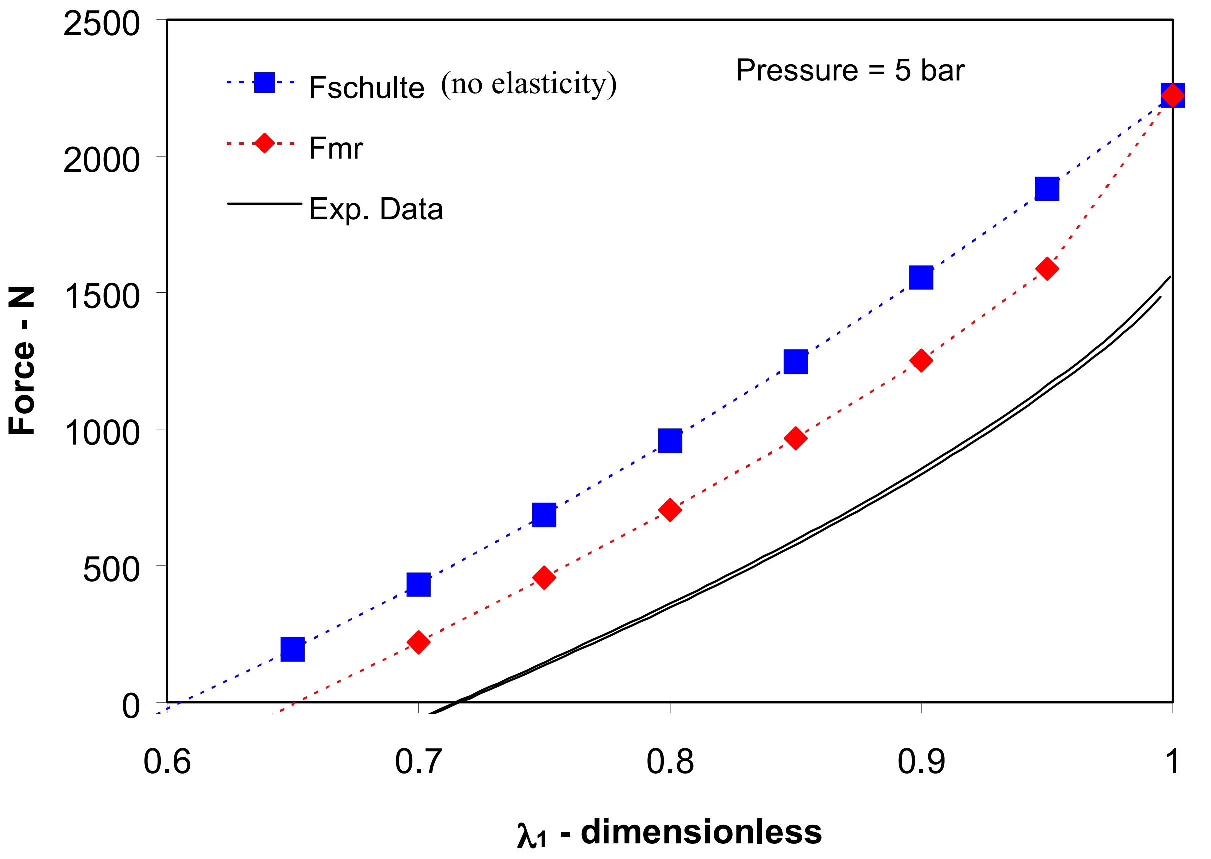
$$W = C_{10}\left(\lambda_1^2+\lambda_2^2+\lambda_3^2-3\right)+C_{01}\left(\lambda_1^2\lambda_2^2+\lambda_1^2\lambda_3^2+\lambda_2^2\lambda_3^2-3\right)$$

$$C_{10} = 118.4 \text{ kPa} \qquad C_{01} = 105.7 \text{ kPa}$$

$$F_{mr} = P\left\{\frac{3(\lambda_1 L_o)^2 - B^2}{4N^2\pi}\right\}$$

$$-V_b\left\{\begin{aligned}&\frac{1}{2L_o^3\lambda_1^3}\left\{4(C_{10}+C_{01})L_o^2\left(-1+\lambda_1^4\right)+\frac{4L_o^6(-1+\lambda_1)\lambda_1^2(1+\lambda_1)\left(C_{10}+C_{01}\lambda_1^2\right)}{\left[-4N^2\pi^2R_o^2+L_o^2\left(-1+\lambda_1^2\right)\right]^2}\right.\\&\left.-\frac{4L_o^4\left(C_{10}+C_{01}\lambda_1^4\right)}{-4N^2\pi^2R_o^2+L_o^2\left(-1+\lambda_1^2\right)}-\frac{L_o^4\lambda_1^4\left[C_{10}+C_{01}\left(-1+2\lambda_1^2\right)\right]}{N^2\pi^2R_o^2}\right\}\end{aligned}\right\}$$

Klute & Hannaford (1998)



Klute & Hannaford (1998)

Corrected Model

$$F = \underbrace{V_0 \frac{dW}{dL} - p \frac{dV_e}{dL}}_{F_{mr}} + F_f$$

Coulombic Friction Term
(fitting parameter)