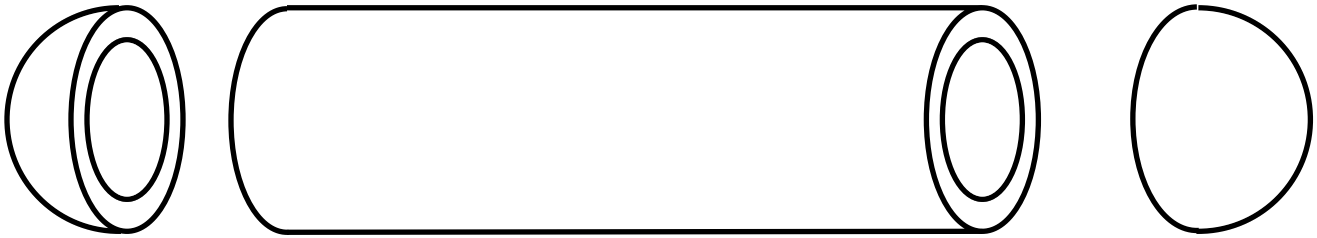
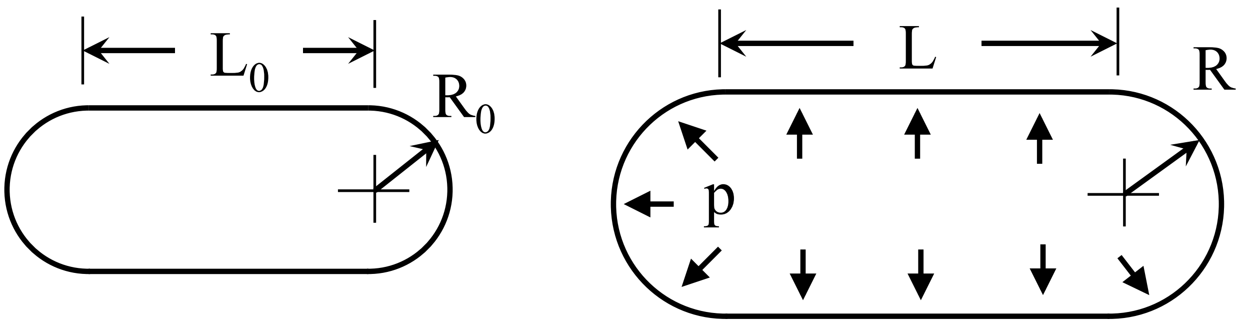
Thin-Walled Tube





Two unknown kinematic variables: R and L

$$\Pi = \Pi\left(R, L\right) \qquad \frac{\partial \Pi}{\partial R} = 0 \quad \frac{\partial \Pi}{\partial L} = 0$$

Cylinder Inflation

For very long tubes, strain energy and air volume in caps/end is *negligible* to energy and volume in the cylinder.

Cylinder: $\lambda_1 = L/L_0$
$\lambda_2 = R/R_0$
$\lambda_3 = 1/\lambda_1\lambda_2 = L_0R_0/LR$

$$W = -\frac{EJ_m}{6}\ln\left(1-\frac{J_1}{J_m}\right) \qquad J_1 = \left(\frac{L}{L_0}\right)^2 + \left(\frac{R}{R_0}\right)^2 + \left(\frac{L_0R_0}{LR}\right)^2 - 3$$

$$V_0 = 2\pi R_0 t_0 L_0 \qquad J_m = \text{given}$$

Enclosed Volume: $V_e = \pi R^2 L$

Total Potential Energy

$$\Pi = WV_0 - pV_e = -\frac{1}{3}\left(\pi R_0 t_0 L_0 E J_m\right)\ln\left(1-\frac{J_1}{J_m}\right) - \pi R^2 L p$$

Solution

$$\frac{\partial \Pi}{\partial R} = \frac{2\pi R_0 t_0 L_0 E}{3\left(1 - J_1/J_m\right)}\left\{\frac{R}{R_0^2} - \frac{L_0^2 R_0^2}{L^2 R^3}\right\} - 2\pi R L p \equiv 0$$

$$\frac{\partial \Pi}{\partial L} = \frac{2\pi R_0 t_0 L_0 E}{3\left(1 - J_1/J_m\right)}\left\{\frac{L}{L_0^2} - \frac{L_0^2 R_0^2}{L^3 R^2}\right\} - \pi R^2 p \equiv 0$$

Non-dimensionalize $\hat{p} = \frac{pR_0}{Et_0}$ $\lambda_1 = L/L_0$ $\lambda_2 = R/R_0$

$$J_1 = \lambda_1^2 + \lambda_2^2 + \lambda_1^{-2}\lambda_2^{-2} - 3 \qquad v = V_e\Big/\left(V_e\right)_0 = \lambda_1\lambda_2^2$$

$$\frac{\partial \Pi}{\partial \lambda_1} = \frac{\partial \Pi}{\partial \lambda_2} = 0 \quad \begin{cases} \dfrac{1}{3\left(1 - J_1/J_m\right)}\left\{\lambda_2 - \dfrac{1}{\lambda_1^2\lambda_2^3}\right\} - \lambda_1\lambda_2\hat{p} = 0 \\[2ex] \dfrac{2}{3\left(1 - J_1/J_m\right)}\left\{\lambda_1 - \dfrac{1}{\lambda_1^3\lambda_2^2}\right\} - \lambda_2^2\hat{p} = 0 \end{cases}$$

Solution

$$\frac{\lambda_2}{3\left(1-J_1/J_m\right)}\left\{\lambda_2-\frac{1}{\lambda_1^2\lambda_2^3}\right\}-\lambda_1\lambda_2^2\hat{p}=0$$

$$-\quad\frac{2\lambda_1}{3\left(1-J_1/J_m\right)}\left\{\lambda_1-\frac{1}{\lambda_1^3\lambda_2^2}\right\}-\lambda_1\lambda_2^2\hat{p}=0$$

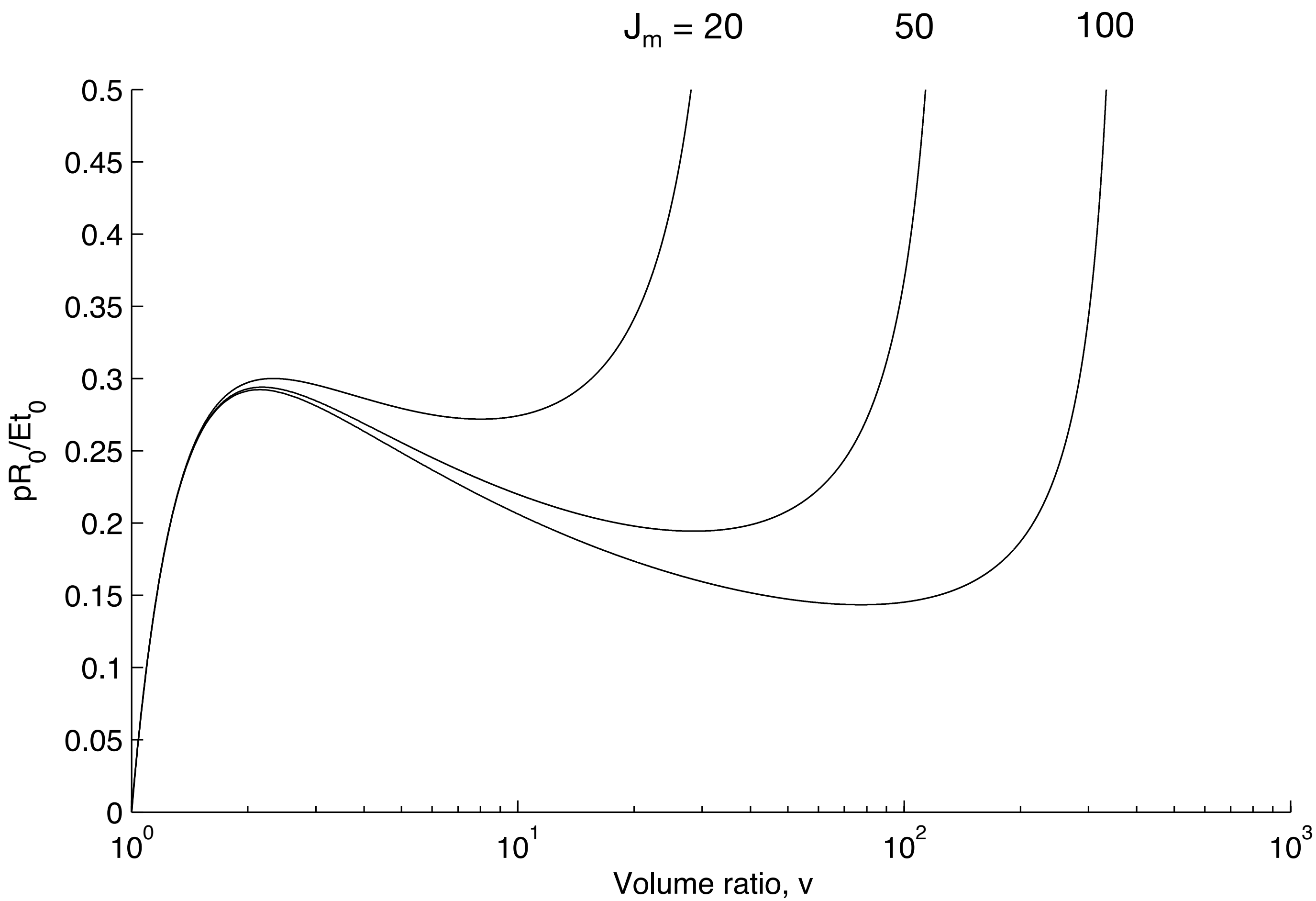
$$\frac{1}{3\left(1-J_1/J_m\right)}\left\{\lambda_2^2-\frac{1}{\lambda_1^2\lambda_2^2}-2\lambda_1^2+\frac{2}{\lambda_1^2\lambda_2^2}\right\}=0$$

$$\left.\begin{aligned}&\Rightarrow \lambda_2^2-2\lambda_1^2+\frac{1}{\lambda_1^2\lambda_2^2}=0\\ &v=\lambda_1\lambda_2^2\end{aligned}\right\}\quad \boxed{\lambda_1=\left\{\frac{1+v^2}{2v}\right\}^{1/3}\qquad \lambda_2=\left\{\frac{2v^4}{1+v^2}\right\}^{1/6}}$$

$$J_1=\lambda_1^2+\lambda_2^2+\lambda_1^{-2}\lambda_2^{-2}-3$$

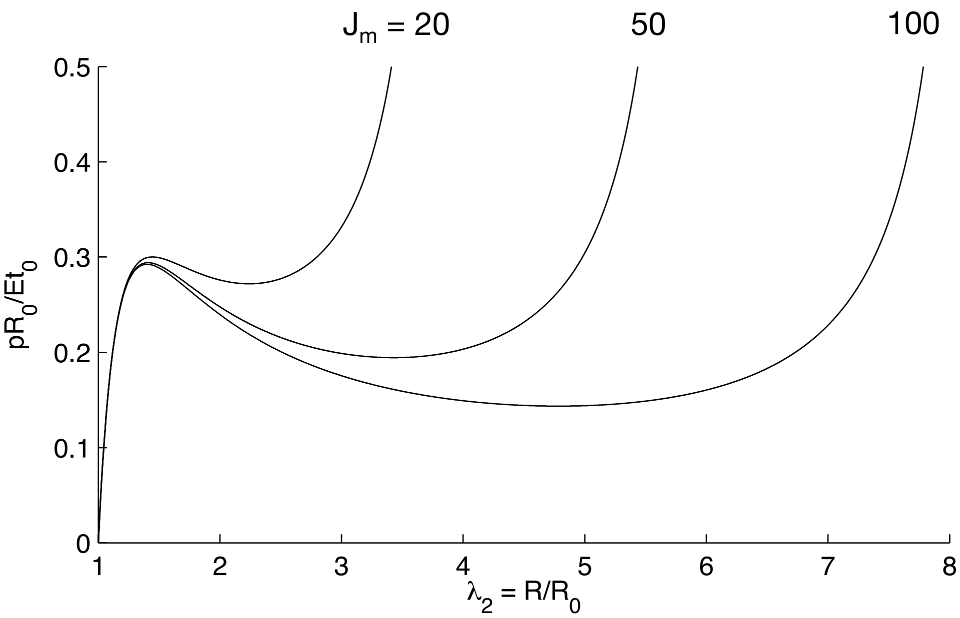
$$\boxed{\hat{p}=\frac{\lambda_2^2}{3v\left(1-J_1/J_m\right)}\left\{1-\frac{1}{v^4}\right\}}$$

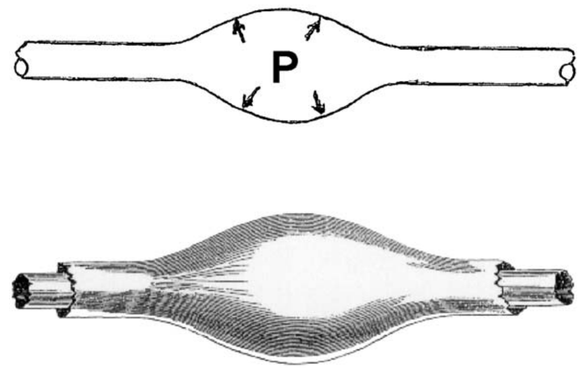
Results



J_m = 20
50
100
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
pR_0/Et_0
10^0
10^1
10^2
10^3
Volume ratio, v

Results





A. N. Gent, "Elastic Instabilities in Rubber" *International Journal of Non-linear Mechanics*, vol. 40, pg. 165-175 (2005).