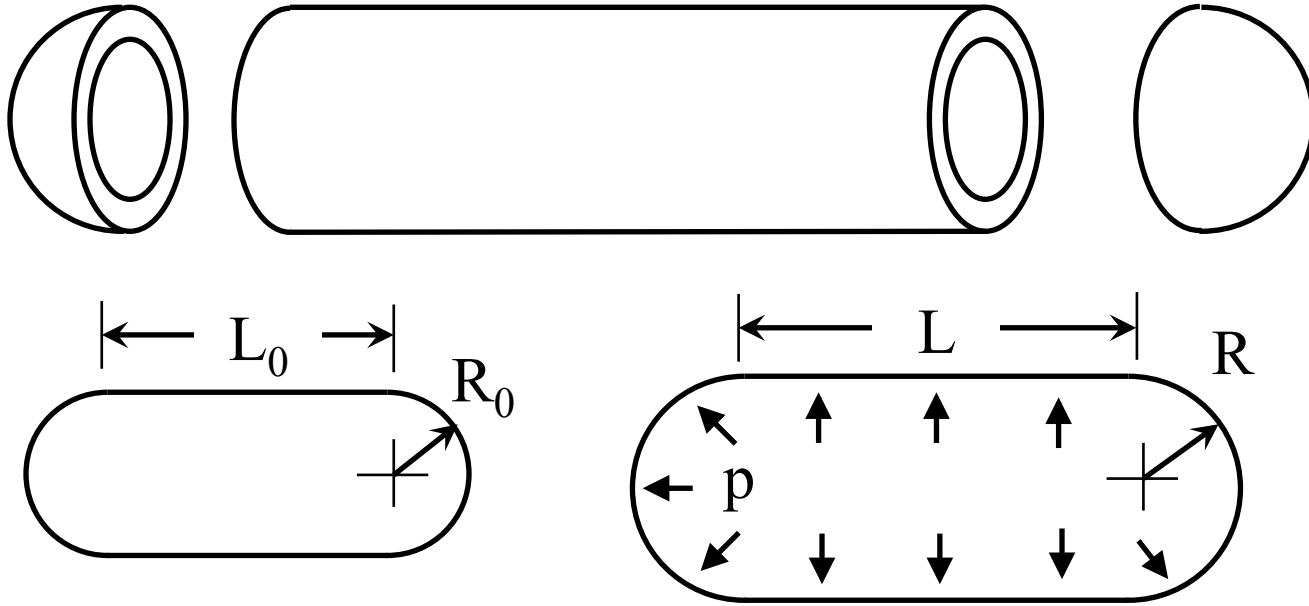


Thin-Walled Tube



Two unknown kinematic variables: R and L

$$\Pi = \Pi(R, L) \quad \frac{\partial \Pi}{\partial R} = 0 \quad \frac{\partial \Pi}{\partial L} = 0$$

Cylinder Inflation

For very long tubes, strain energy and air volume in caps/end is *negligible* to energy and volume in the cylinder.

Cylinder:

$$\lambda_1 = L/L_0$$
$$\lambda_2 = R/R_0$$
$$\lambda_3 = 1/\lambda_1\lambda_2 = L_0R_0/LR$$

$$W = -\frac{EJ_m}{6} \ln\left(1 - \frac{J_1}{J_m}\right) \quad J_1 = \left(\frac{L}{L_0}\right)^2 + \left(\frac{R}{R_0}\right)^2 + \left(\frac{L_0R_0}{LR}\right)^2 - 3$$

$$V_0 = 2\pi R_0 t_0 L_0 \quad J_m = \text{given}$$

Enclosed Volume: $V_e = \pi R^2 L$

Total Potential Energy

$$\Pi = WV_0 - pV_e = -\frac{1}{3}(\pi R_0 t_0 L_0 EJ_m) \ln\left(1 - \frac{J_1}{J_m}\right) - \pi R^2 L p$$

Solution

$$\frac{\partial \Pi}{\partial R} = \frac{2\pi R_0 t_0 L_0 E}{3(1 - J_1/J_m)} \left\{ \frac{R}{R_0^2} - \frac{L_0^2 R_0^2}{L^2 R^3} \right\} - 2\pi R L p \equiv 0$$

$$\frac{\partial \Pi}{\partial L} = \frac{2\pi R_0 t_0 L_0 E}{3(1 - J_1/J_m)} \left\{ \frac{L}{L_0^2} - \frac{L_0^2 R_0^2}{L^3 R^2} \right\} - \pi R^2 p \equiv 0$$

Non-dimensionalize $\hat{p} = \frac{p R_0}{E t_0} \quad \lambda_1 = L/L_0 \quad \lambda_2 = R/R_0$

$$J_1 = \lambda_1^2 + \lambda_2^2 + \lambda_1^{-2} \lambda_2^{-2} - 3 \quad v = V_e / (V_e)_0 = \lambda_1 \lambda_2^2$$

$$\frac{\partial \Pi}{\partial \lambda_1} = \frac{\partial \Pi}{\partial \lambda_2} = 0 \quad \left\{ \begin{array}{l} \frac{1}{3(1 - J_1/J_m)} \left\{ \lambda_2 - \frac{1}{\lambda_1^2 \lambda_2^3} \right\} - \lambda_1 \lambda_2 \hat{p} = 0 \\ \frac{2}{3(1 - J_1/J_m)} \left\{ \lambda_1 - \frac{1}{\lambda_1^3 \lambda_2^2} \right\} - \lambda_2^2 \hat{p} = 0 \end{array} \right.$$

Solution

$$\frac{\lambda_2}{3(1-J_1/J_m)} \left\{ \lambda_2 - \frac{1}{\lambda_1^2 \lambda_2^3} \right\} - \lambda_1 \lambda_2^2 \hat{p} = 0$$

$$- \frac{2\lambda_1}{3(1-J_1/J_m)} \left\{ \lambda_1 - \frac{1}{\lambda_1^3 \lambda_2^2} \right\} - \lambda_1 \lambda_2^2 \hat{p} = 0$$

$$\frac{1}{3(1-J_1/J_m)} \left\{ \lambda_2^2 - \frac{1}{\lambda_1^2 \lambda_2^2} - 2\lambda_1^2 + \frac{2}{\lambda_1^2 \lambda_2^2} \right\} = 0$$

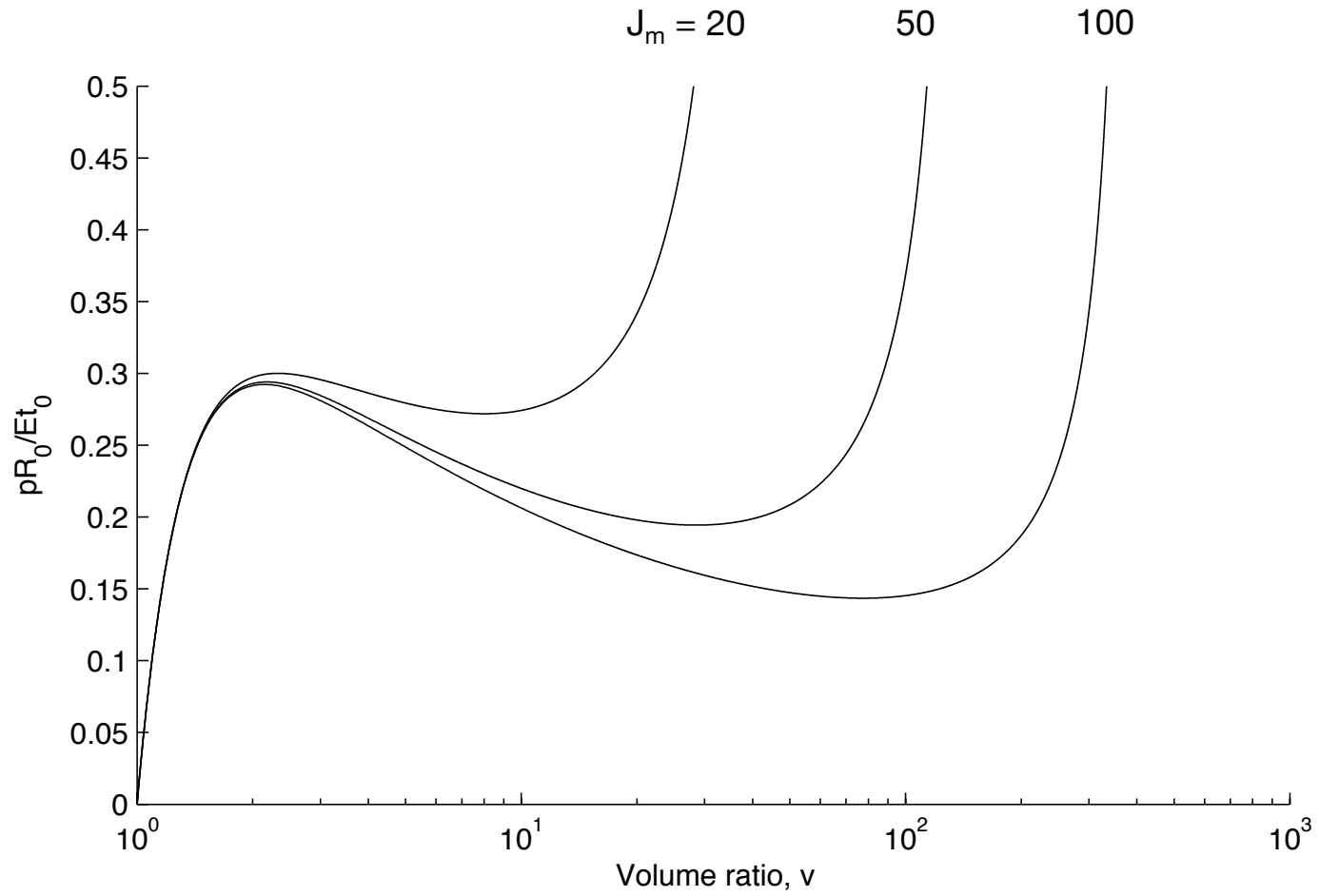
$$\Rightarrow \lambda_2^2 - 2\lambda_1^2 + \frac{1}{\lambda_1^2 \lambda_2^2} = 0$$
$$v = \lambda_1 \lambda_2^2$$

$$\left. \vphantom{\Rightarrow} \right\} \boxed{\lambda_1 = \left\{ \frac{1+v^2}{2v} \right\}^{1/3} \quad \lambda_2 = \left\{ \frac{2v^4}{1+v^2} \right\}^{1/6}}$$

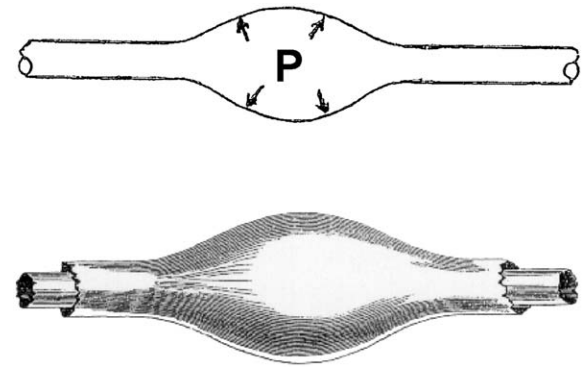
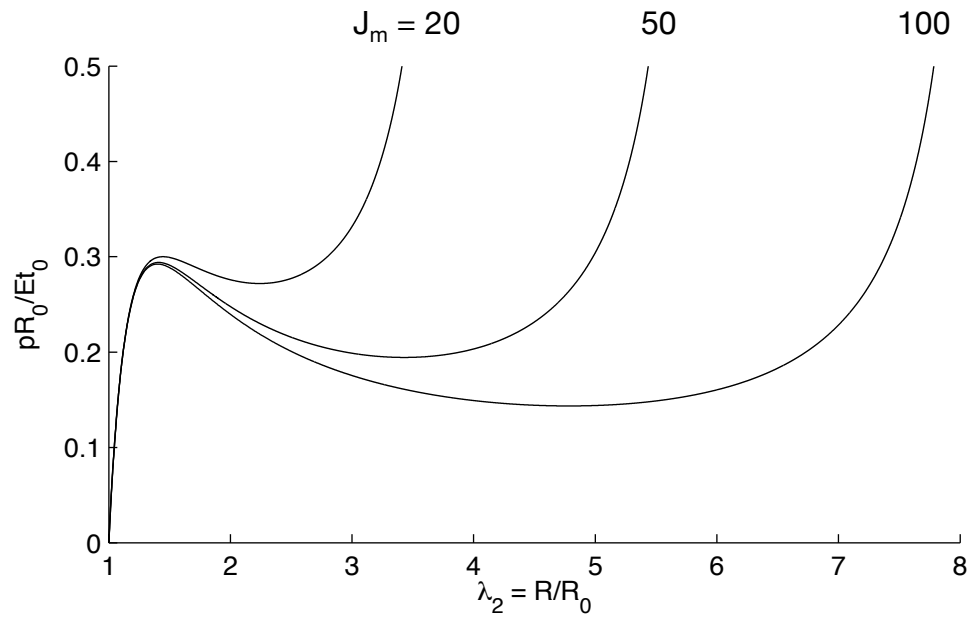
$$J_1 = \lambda_1^2 + \lambda_2^2 + \lambda_1^{-2} \lambda_2^{-2} - 3$$

$$\boxed{\hat{p} = \frac{\lambda_2^2}{3v(1-J_1/J_m)} \left\{ 1 - \frac{1}{v^4} \right\}}$$

Results



Results



A. N. Gent, "Elastic Instabilities in Rubber"
*International Journal of Non-linear
Mechanics*, vol. 40, pg. 165-175 (2005).