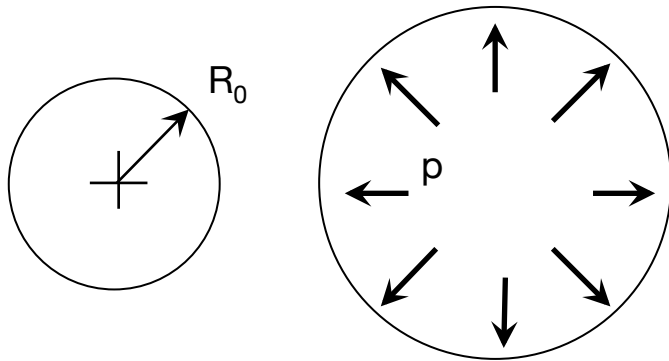


Sphere Inflation



Equibiaxial Loading:

$$\lambda_1 = \lambda_2 = \lambda \equiv \frac{R}{R_0} \quad \lambda_3 = \frac{1}{\lambda_1 \lambda_2} = \frac{1}{\lambda^2}$$

$$W = W(\lambda_1, \lambda_2, \lambda_3)$$

$$\Pi = \Pi(R) = 4\pi R_0^2 t_0 W - \frac{4}{3} \pi R^3 p$$

$$\frac{d\Pi}{dR} = 0 \Rightarrow \boxed{p = \frac{t_0 R_0^2}{R^2} \frac{dW}{dR}}$$

Constitutive Laws

NeoHookean

$$W = \frac{E}{6} (\lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3)$$

“1-param” Ogden

$$W = \frac{E}{24} (\lambda_1^4 + \lambda_2^4 + \lambda_3^4 - 3)$$

“Gent” Solid

$$W = -\frac{EJ_m}{6} \ln \left(1 - \frac{J_1}{J_m} \right)$$

$$J_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3$$

J_m = maximum possible value of J_1
due to strain hardening

Equibiaxial Loading

$$\sigma_1 = \sigma_2 = \sigma \quad \sigma_3 = 0 \quad \lambda_1 = \lambda_2 = \lambda \quad \lambda_3 = \frac{1}{\lambda_1 \lambda_2} = \frac{1}{\lambda^2}$$

$$\sigma_i = \lambda_i \frac{dW}{d\lambda_i} - p \Rightarrow \sigma = \left[\lambda_1 \frac{dW}{d\lambda_1} - \lambda_3 \frac{dW}{d\lambda_3} \right]_{\lambda_1=\lambda_2=\lambda, \lambda_3=\lambda^{-2}}$$

NeoHookean

$$\sigma = \frac{E}{3} \left(\lambda^2 - \frac{1}{\lambda^4} \right)$$

“1-param” Ogden

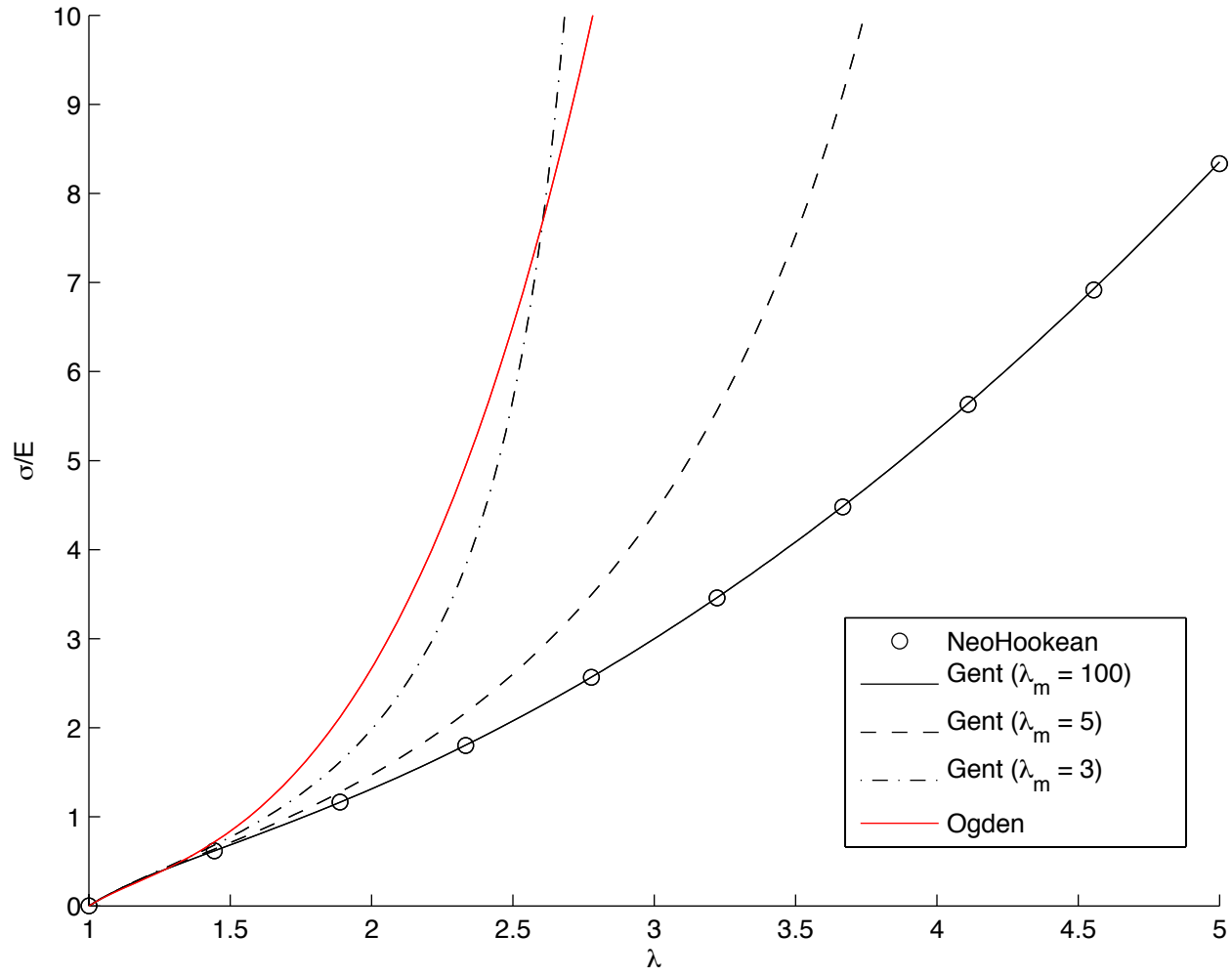
$$\sigma = \frac{E}{6} \left(\lambda^4 - \frac{1}{\lambda^8} \right)$$

“Gent” Solid

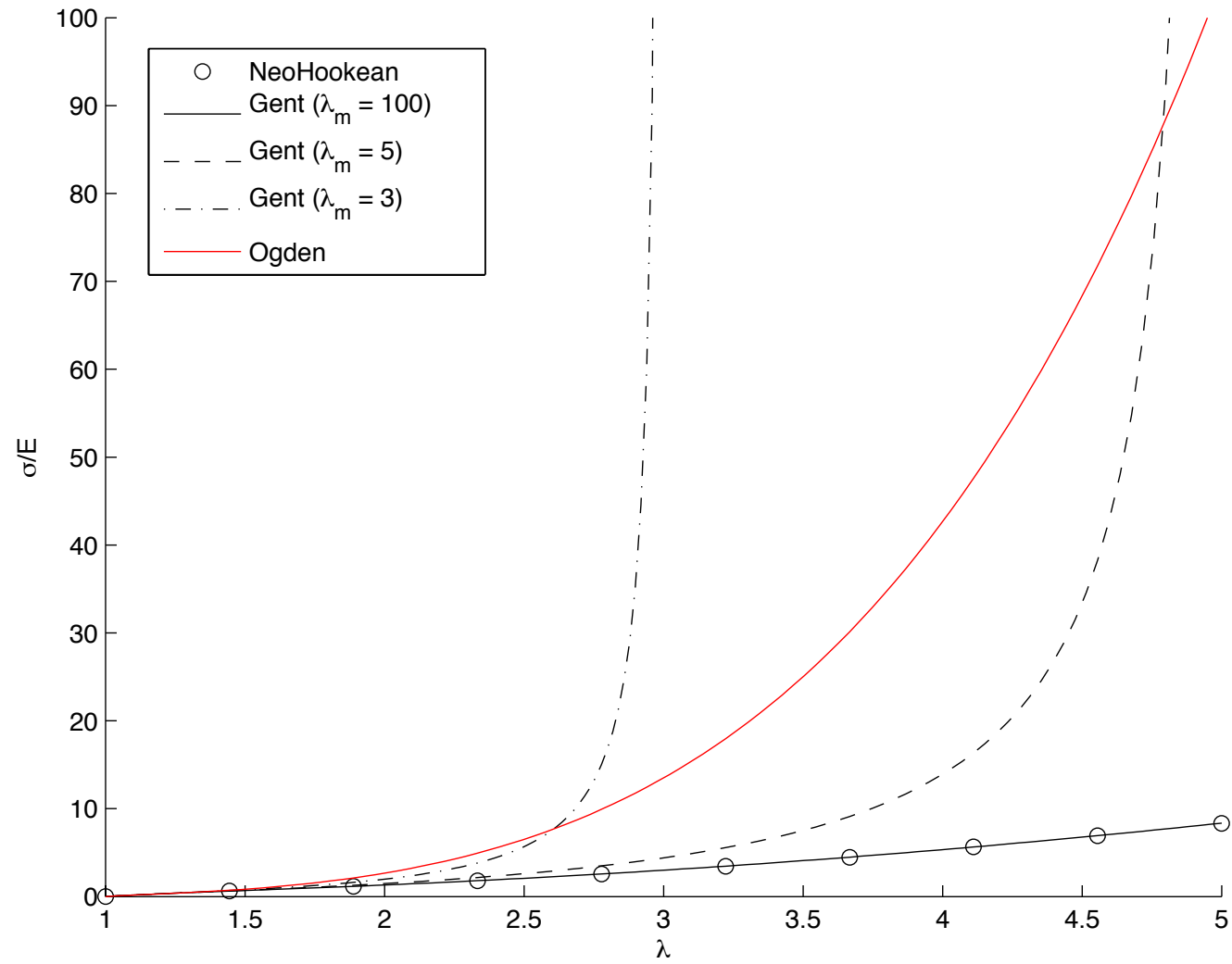
$$\sigma = \frac{E}{3(1 - J_1/J_m)} \left\{ \lambda^2 - \frac{1}{\lambda^4} \right\}$$

$$\left| \begin{array}{l} J_1 = 2\lambda^2 + \lambda^{-4} - 3 \\ J_m = 2\lambda_m^2 + \lambda_m^{-4} - 3 \end{array} \right.$$

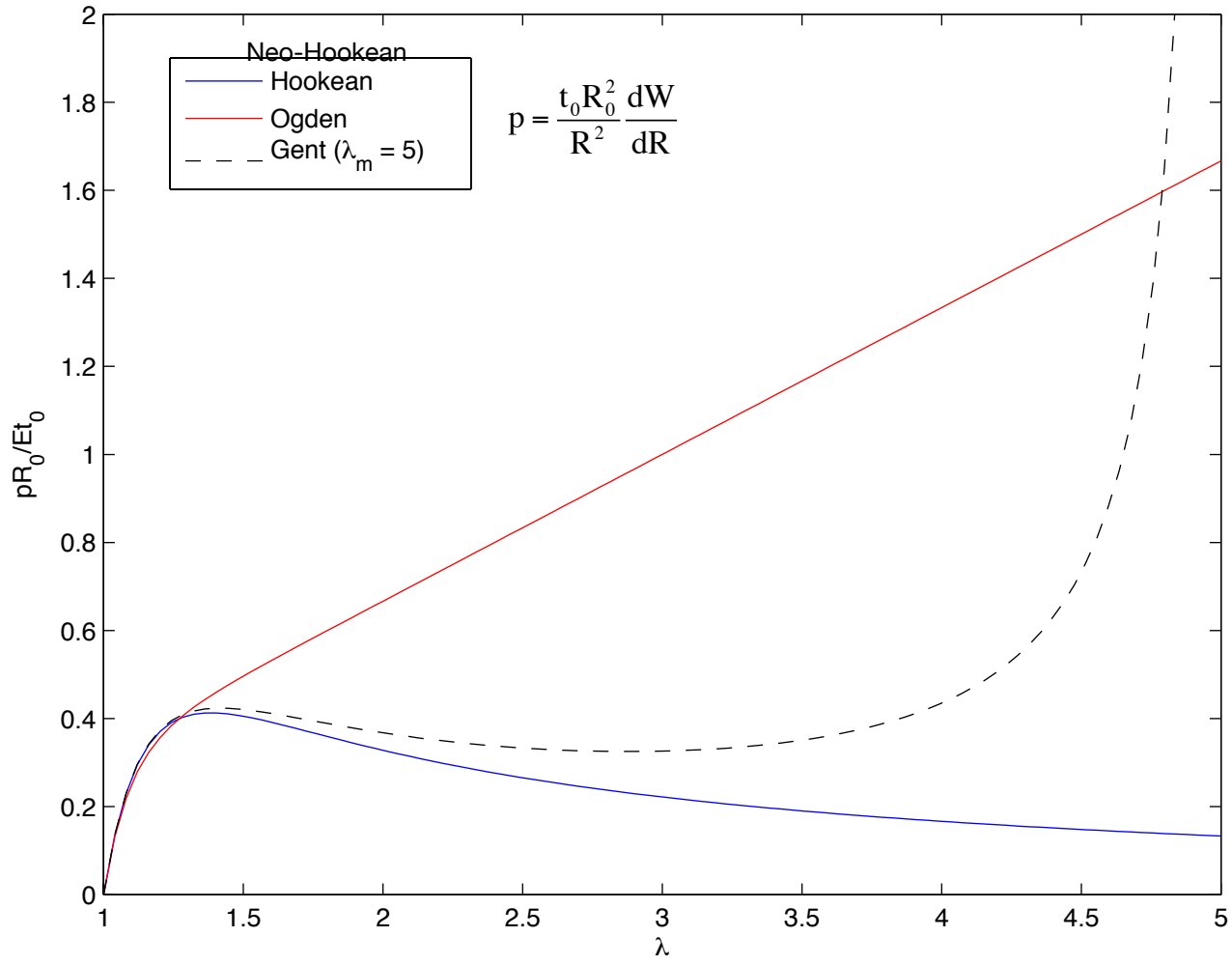
Stress-Stretch



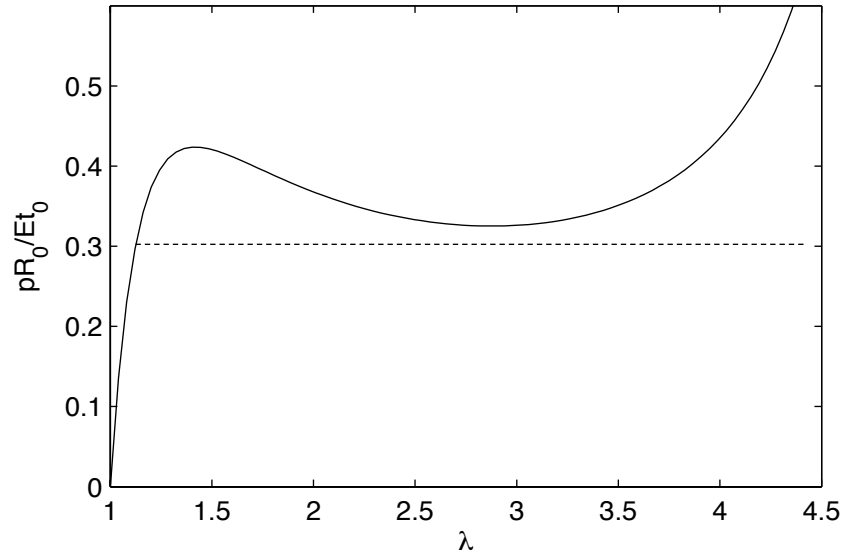
Stress-Stretch



Solution

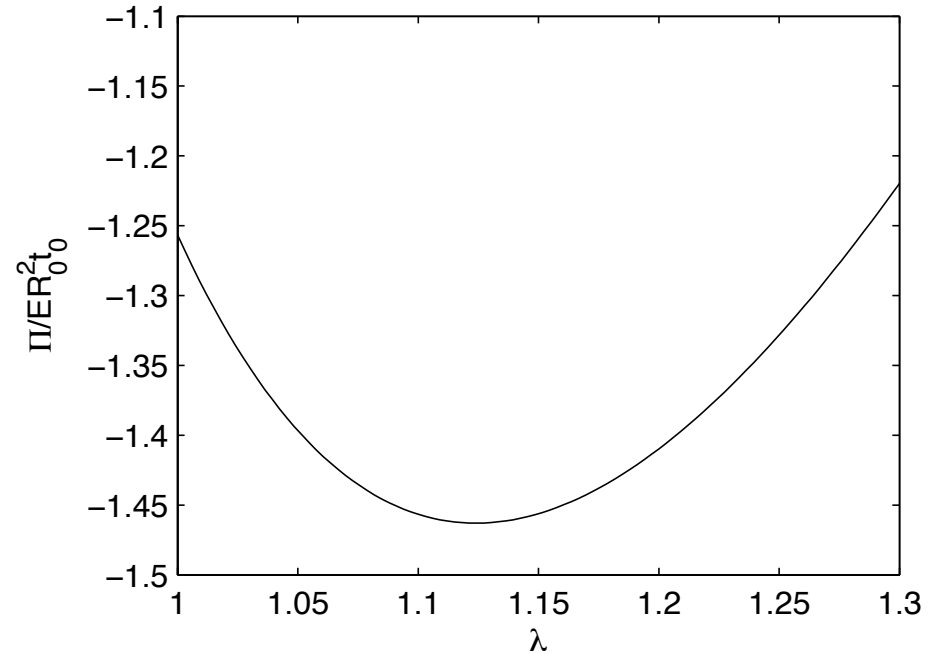


Gent Balloon ($\lambda_m = 5$)

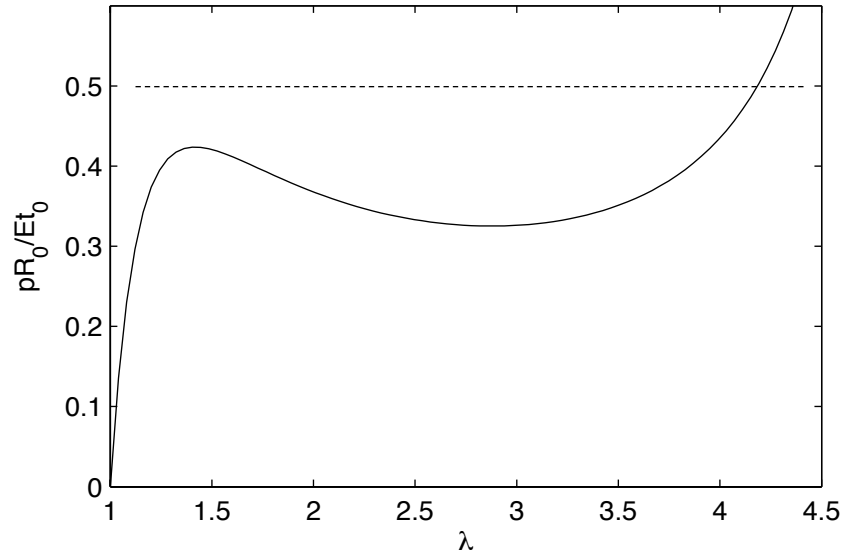


$$\hat{\Pi} = \frac{\Pi}{R_0^2 t_0 E} = -\frac{2\pi J_m}{3} \ln\left(1 - \frac{J_1}{J_m}\right) - \frac{4}{3} \pi \lambda^3 \hat{p}$$

$$\hat{p} = \frac{pR_0}{Et_0}$$

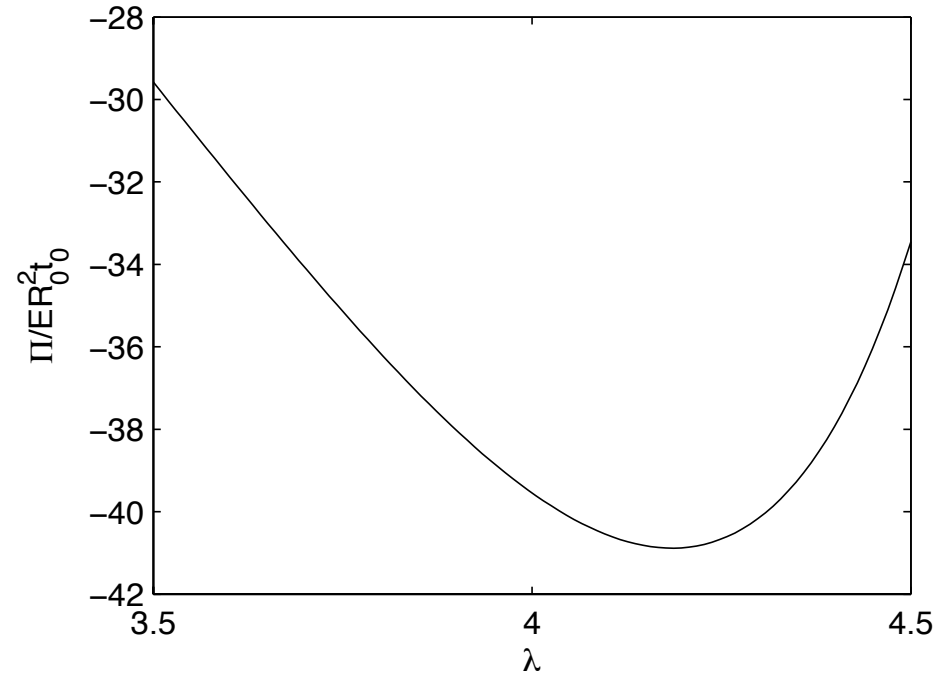


Gent Balloon ($\lambda_m = 5$)

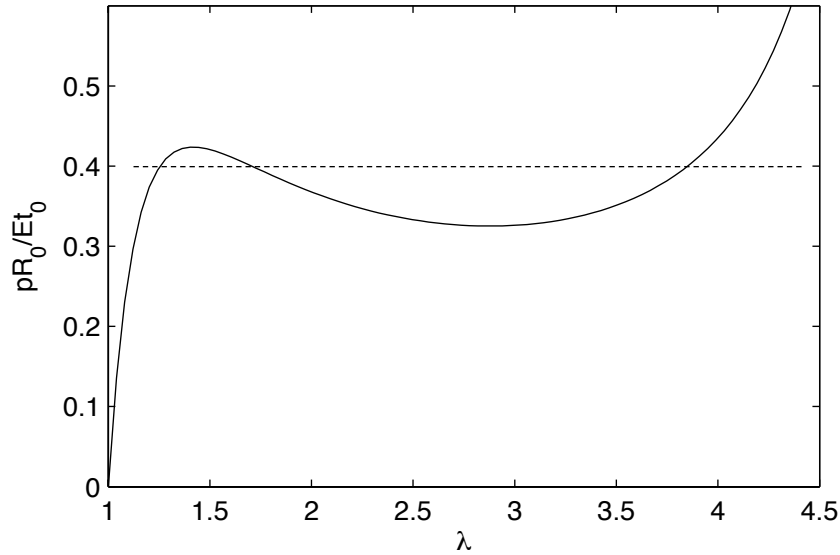


$$\hat{\Pi} = \frac{\Pi}{R_0^2 t_0 E} = -\frac{2\pi J_m}{3} \ln\left(1 - \frac{J_1}{J_m}\right) - \frac{4}{3} \pi \lambda^3 \hat{p}$$

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Gent Balloon ($\lambda_m = 5$)



$$\hat{\Pi} = \frac{\Pi}{R_0^2 t_0 E} = -\frac{2\pi J_m}{3} \ln\left(1 - \frac{J_1}{J_m}\right) - \frac{4}{3} \pi \lambda^3 \hat{p}$$

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