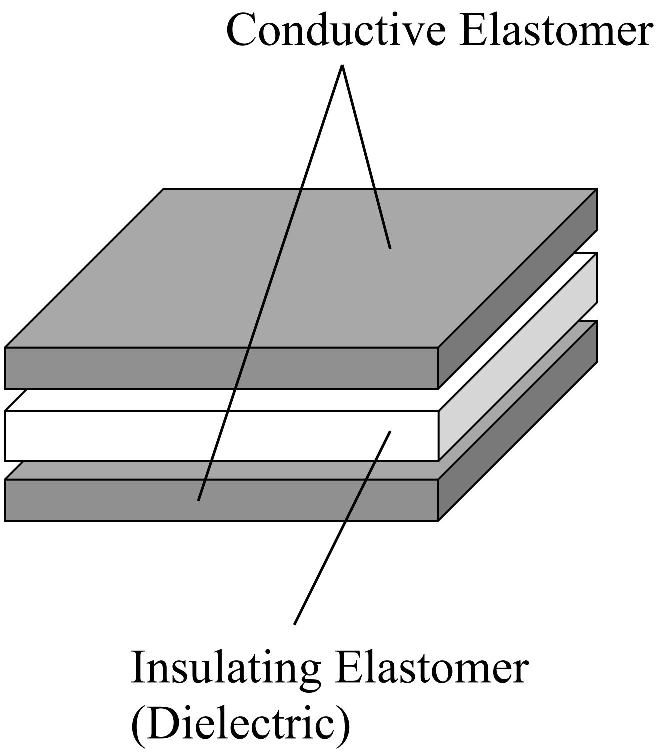
Dielectric Elastomers



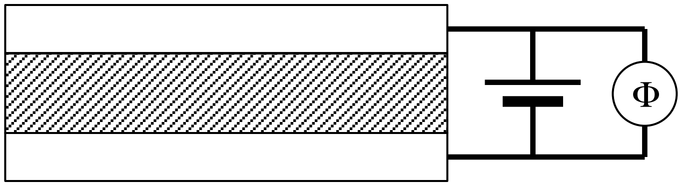
"Soft" Parallel Plate Capacitor

- **Sensor** – capacitance changes w/ pressure & stretch
- **Actuator** – thickness decreases (area increases) under applied electrostatic pressure ("Maxwell Stress")
- **Generator/Transducer** – elastic deformation changes electrostatic potential of surface charge

Implementation

- **Circular Membrane** – prestretch and bond to a rigid ring
- **Flexural Structure** – prestretch capacitor and bond to a flexible substrate
- **Stacked Capacitor** – no prestretch; requires IPN treatment to enhance dielectric breakdown strength
- **Balloon** – thin-walled spherical capacitor; inflate with air
- **Spring-roll** – thin-walled cylindrical capacitor; prestretch and rolled around a helical spring

Parallel-Plate Capacitor



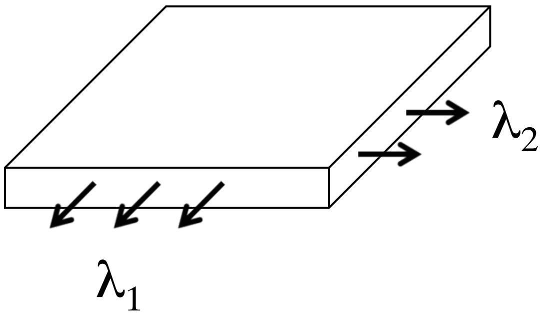
$$q = C\Phi \qquad C = \frac{\varepsilon A}{t}$$

units: F = “Farad” = N-m/V^2
Electric permittivity: $\varepsilon = \varepsilon_r \varepsilon_0$
$\varepsilon_0 = 8.85 \times 10^{-12}$ F/m = vacuum permittivity
ε_r = relative permittivity (“dielectric constant”)
e.g. air ~ 1; rubber ~ 2-5; silicon ~ 11; water ~ 100; PZT ~ 1000

<u>Analogy</u>
$\Phi = C^{-1}q \sim F = kx$
$\Phi \sim F$, $C \sim k^{-1}$, $q \sim x$

Electric displacement: $D = q/A$
Electric Field: $E = \Phi/t$
Maxwell’s equation: $D = \varepsilon E$

Capacitive Sensing



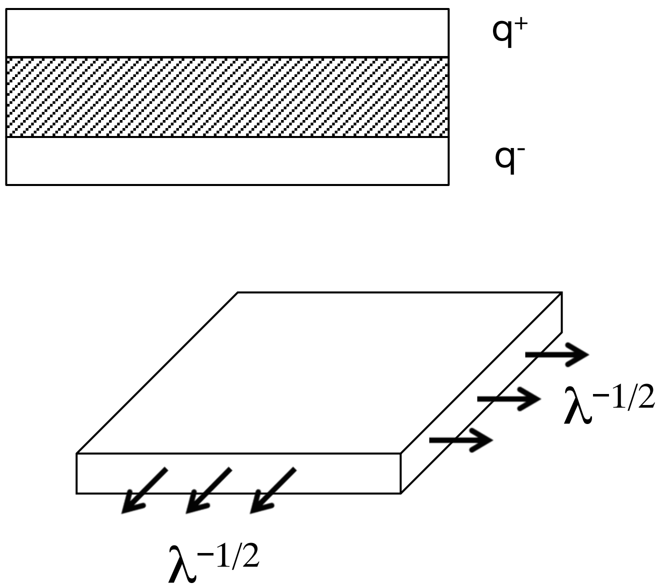
$$C = \frac{\varepsilon_r \varepsilon_0 A_0}{t_0}\left(\frac{\lambda_1 \lambda_2}{\lambda_3}\right) \Rightarrow \frac{C}{C_0} = \frac{\lambda_1 \lambda_2}{\lambda_3}$$

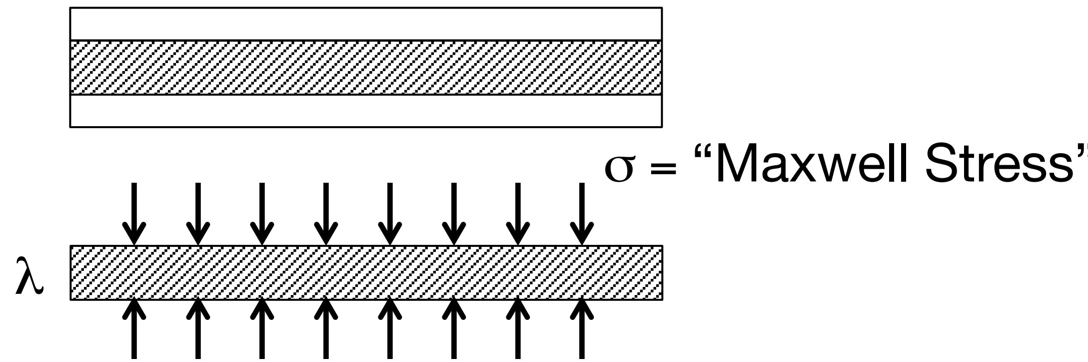
Uniaxial strain: $\lambda_1 = \lambda, \lambda_2 = \lambda_3 = \lambda^{-1/2} \Rightarrow C/C_0 = \lambda$

Biaxial strain: $\lambda_1 = \lambda_2 = \lambda, \lambda_3 = \lambda^{-2} \Rightarrow C/C_0 = \lambda^4$

Pressure/squeezing: $\lambda_3 = \lambda, \lambda_1 = \lambda_2 = \lambda^{-1/2} \Rightarrow C/C_0 = \lambda^{-2}$

Maxwell Stress





Equibiaxial loading: $\lambda_1 = \lambda_2 = \lambda^{-2}$, $\lambda_3 = \lambda$

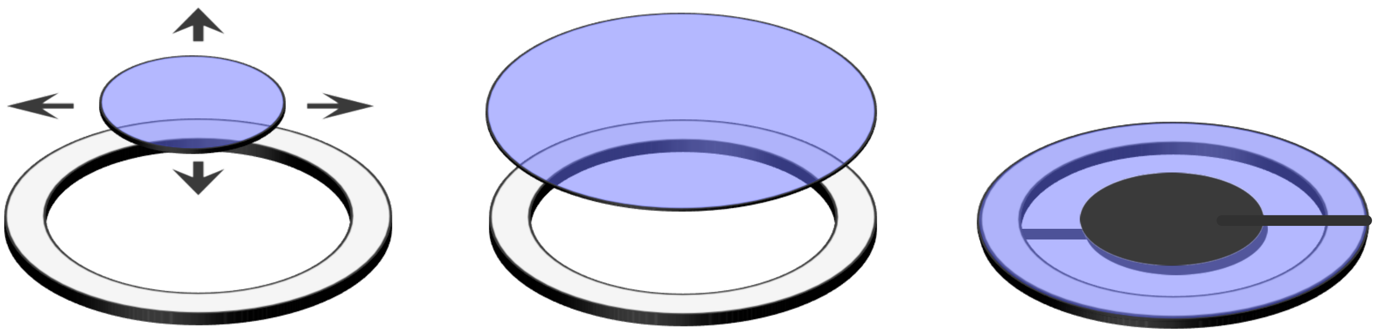
Prescribed surface charge: q

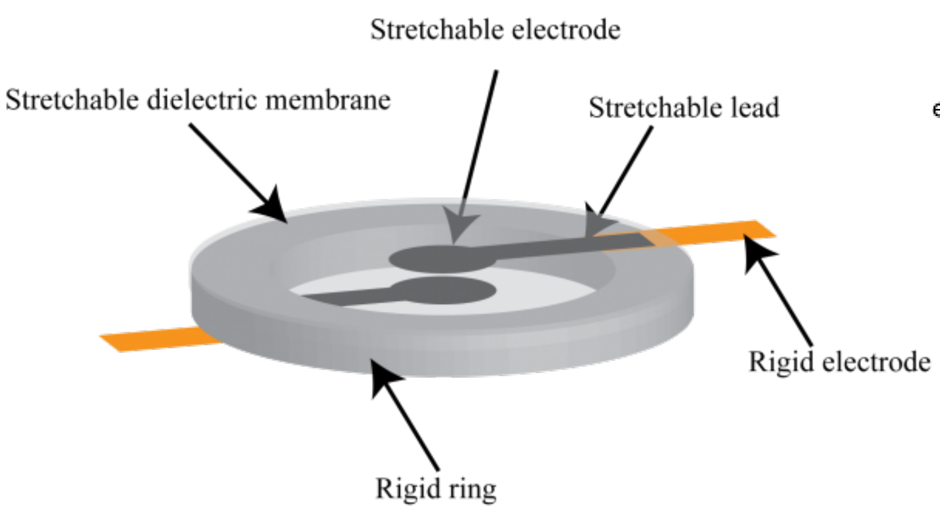
Voltage: $\Phi = q/C$

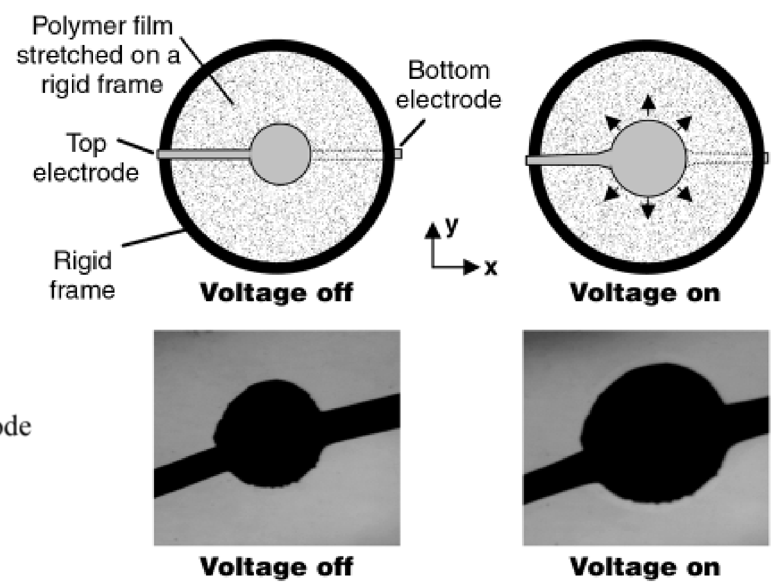
Maxwell Stress: $\sigma = qE/A$

High-speed electrically actuated elastomers with strain greater than 100%
Ron Pelrine, Roy Konbluh, Qibing Pei, Jose Joseph, *Science* vol. 287 (2000)

1739 citations (as of 2/17/16)







Stress-Strain Method

$$W = \frac{\mu}{4}\left(\lambda_1^4 + \lambda_2^4 + \lambda_3^4 - 3\right)$$

$$\sigma_1 = \sigma_2 = 0 = \frac{\mu}{\lambda^2} - p \Rightarrow p = \frac{\mu}{\lambda^2}$$

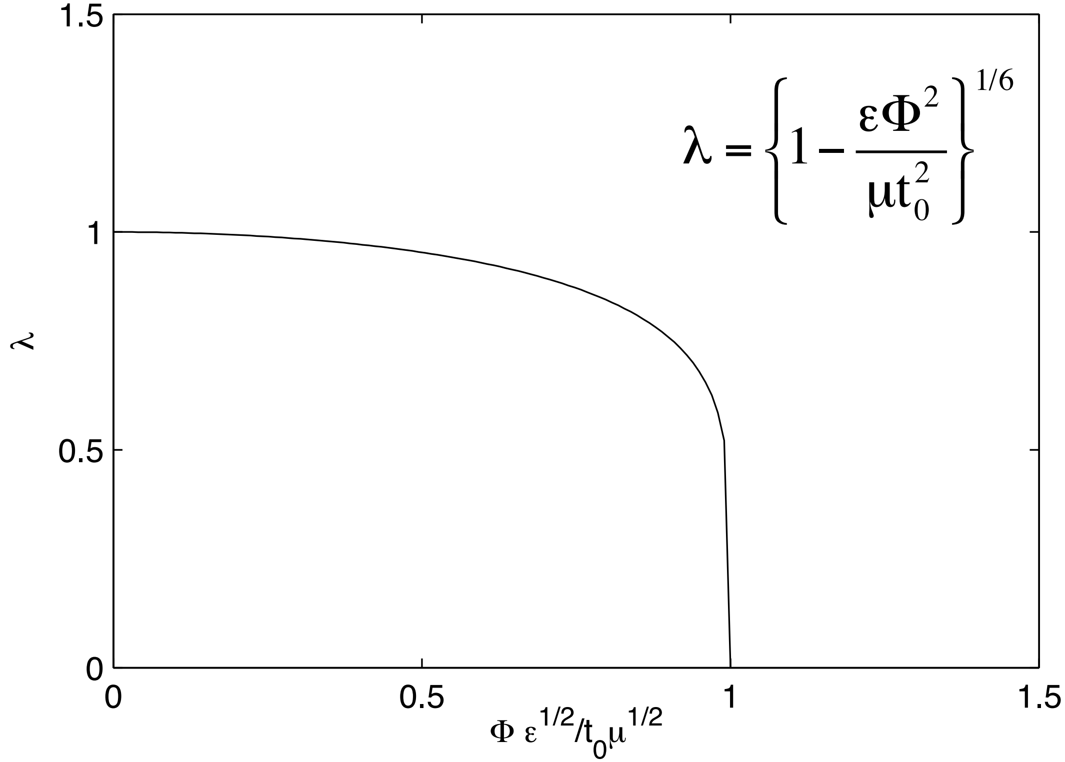
$$\sigma_3 = \mu\lambda^4 - p = \mu\left(\lambda^4 - \frac{1}{\lambda^2}\right) \equiv -\frac{qE}{A}$$

$$\left\{ \begin{aligned} q &= \frac{\varepsilon A_0 \Phi}{\lambda^2 t_0} \\ E &= \frac{\Phi}{t} = \frac{\Phi}{\lambda t_0} \\ A &= \frac{A_0}{\lambda} \end{aligned} \right.$$

$$\therefore \lambda = \left\{1 - \frac{\varepsilon\Phi^2}{\mu t_0^2}\right\}^{1/6} \Leftarrow \mu\left(\lambda^4 - \frac{1}{\lambda^2}\right) = -\frac{\varepsilon\Phi^2}{\lambda^2 t_0^2}$$

$$\lambda = \left\{1 - \hat{\Phi}^2\right\}^{1/6} \quad \text{where,} \quad \hat{\Phi} = \Phi\left\{\frac{1}{t_0}\sqrt{\frac{\varepsilon}{\mu}}\right\}$$

Voltage limited by *electromechanical instability* when $\Phi = \Phi_{cr} := t_0\sqrt{\frac{\mu}{\varepsilon}}$



Typical values:

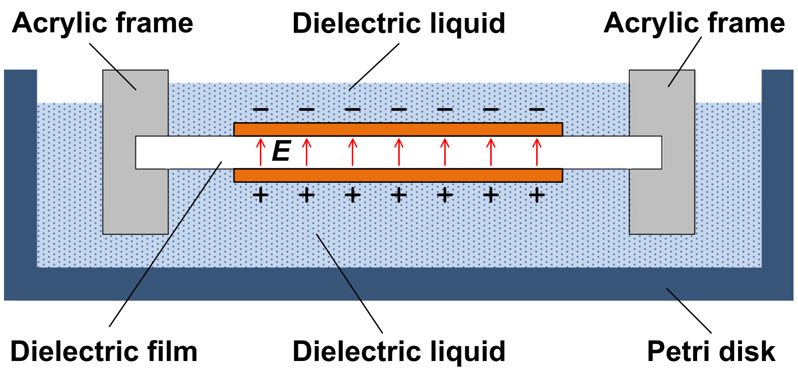
- $t_0 \sim 10^{-4}$ m
- $\mu \sim 10^{5}$ Pa
- $\varepsilon \sim 10^{-11}$ F/m

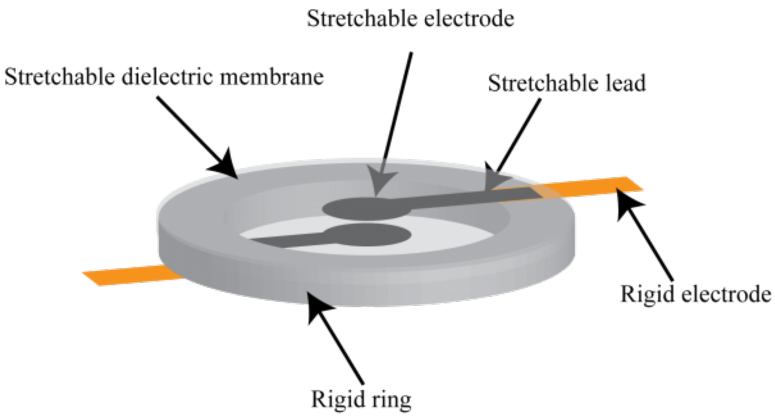
$$\Phi_{cr} = t_0\sqrt{\frac{\mu}{\varepsilon}} \sim 10^{-4}\sqrt{\frac{10^5}{10^{-11}}} \sim 10^4\,\mathrm{V}$$

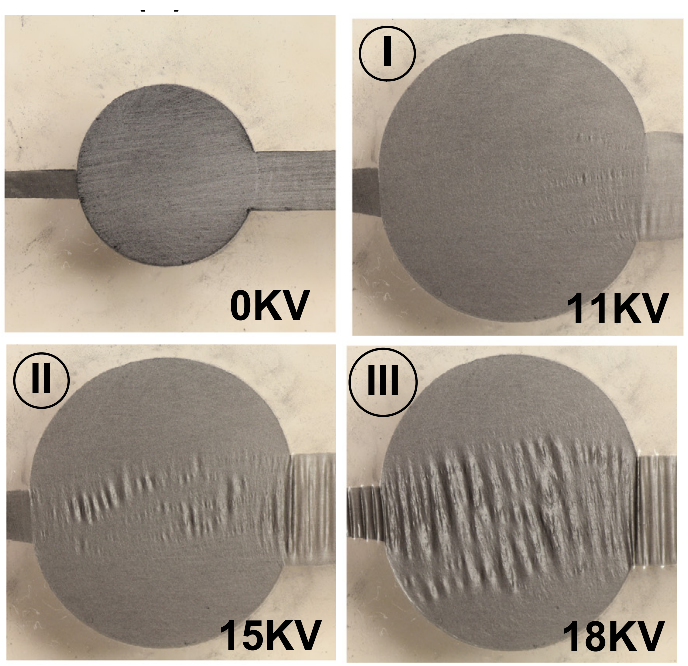
Electromechanical Instability (Pull-In)

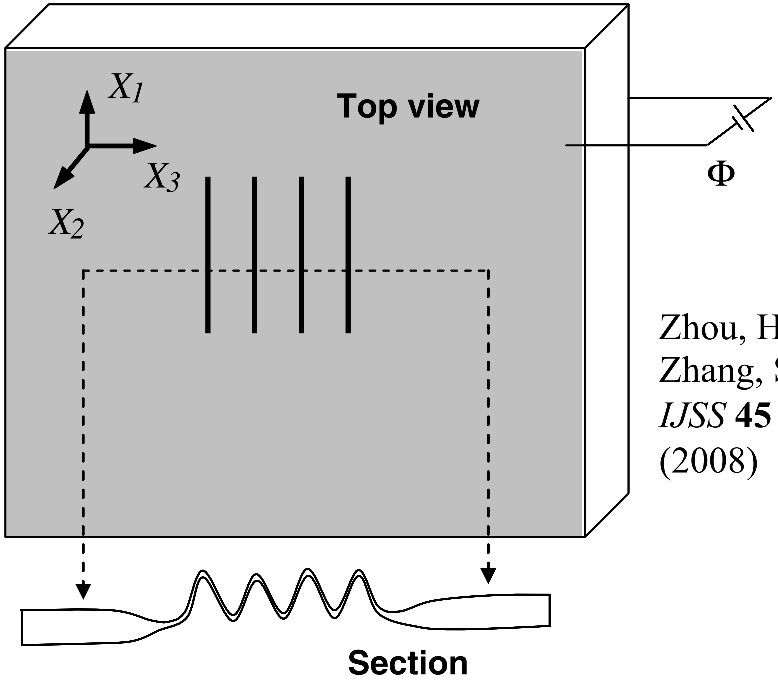
Very high dielectric strength for dielectric elastomer actuators in liquid dielectric immersion

T-G La and G-K Lau *APL* **102** 192905 (2013)





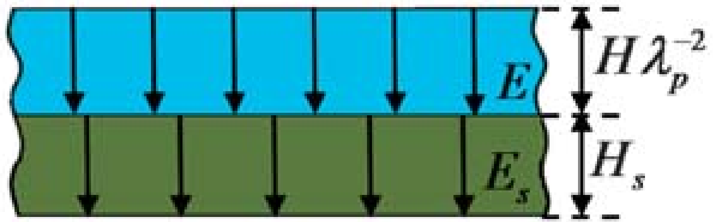


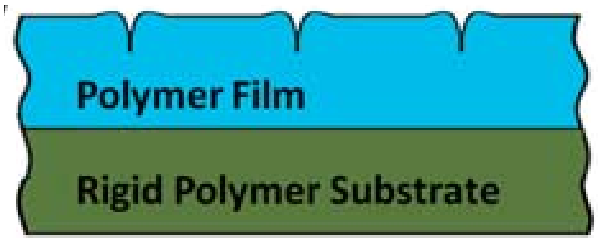


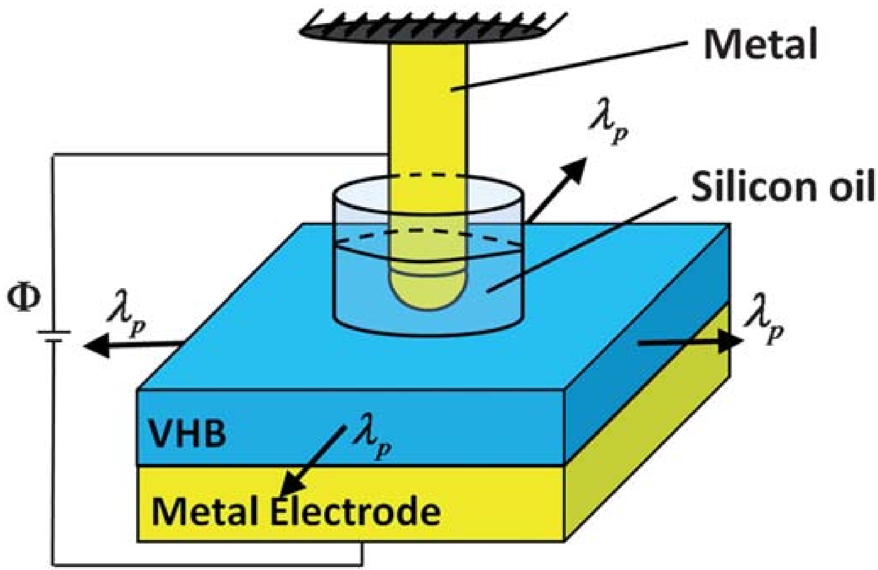
Zhou, Hong, Zhao, Zhang, Suo, *IJSS* **45** 3739-3750 (2008)

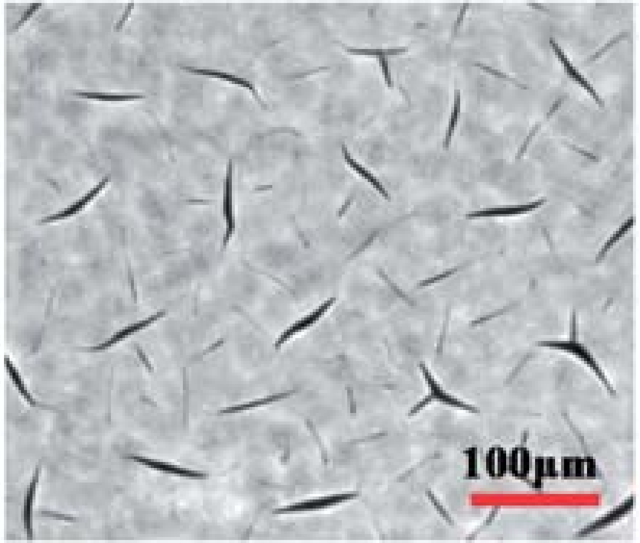
Electro-creasing instability in deformed polymers: experiment and theory

Q. Wang, M. Tahir, L. Zhang, X. Zhao *Soft Matter* **7** 6583 (2011)





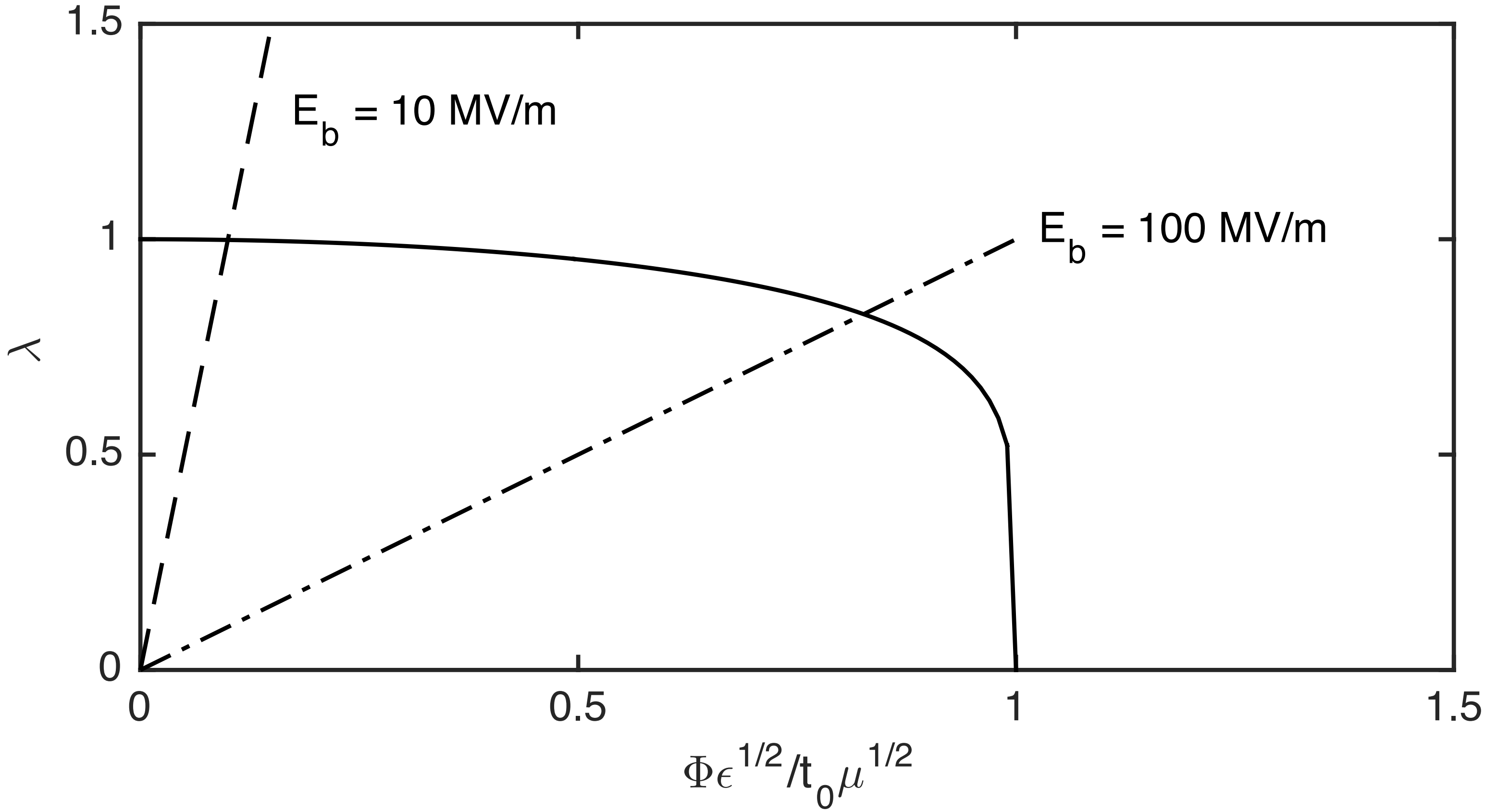




The voltage is also limited by *electrical breakdown*:

- Internal electric field is so large that bound charge carriers get energized and mobilized
- The charges (electrons or ions) crash into neighbors and form a cascade of interactions
- Excited charges carry current through the dielectric and cause an electrical discharge (much like lightening in air)
- Dielectric momentarily acts like a conductor – can lead to permanent damage
- To prevent breakdown, E must remain less than the breakdown strength E_b

$$E = \frac{\Phi}{\lambda t_0} = \frac{\hat{\Phi}}{\lambda}\sqrt{\frac{\mu}{\varepsilon}} < E_b \Rightarrow \boxed{\lambda > \frac{\hat{\Phi}}{E_b}\sqrt{\frac{\mu}{\varepsilon}}}$$

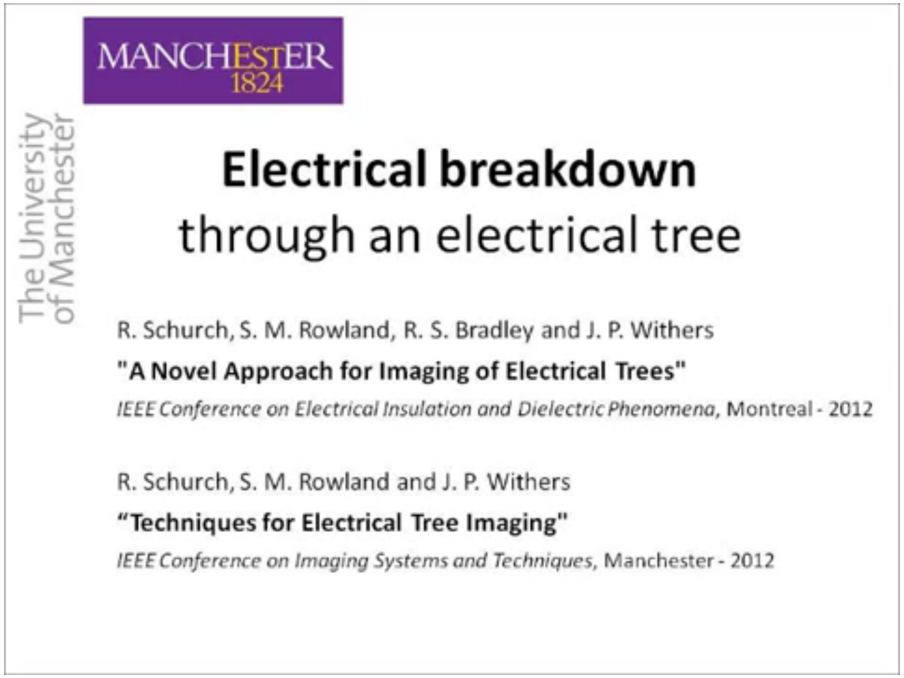


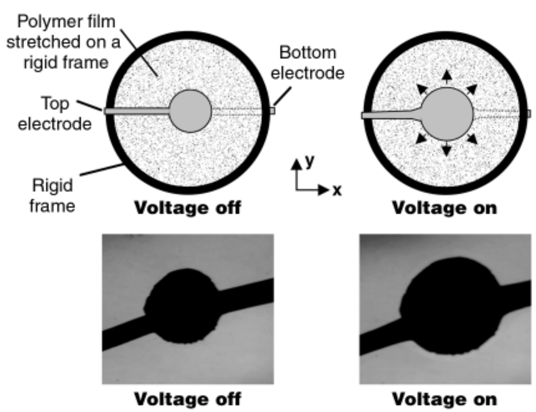
Typical values:

- $\mu \sim 10^5$ Pa
- $\varepsilon \sim 10^{-11}$ F/m
- $E_b \sim 10^7$-10^8 MV/m

$$\lambda > \frac{\hat{\Phi}}{E_b}\sqrt{\frac{\mu}{\varepsilon}} \sim \left\{\frac{1}{10^8}\sqrt{\frac{10^5}{10^{-11}}}\right\}\hat{\Phi} \sim \hat{\Phi}$$

Electrical Breakdown in Dielectrics





Material	Prestrain (*x*,*y*) (%)	Actuated relative thickness strain (%)	Actuated relative area strain (%)	Field strength (MV/m)
		Circular strain		
HS3 silicone	(68,68)	48	93	110
	(14,14)	41	69	72
CF19-2186 silicone	(45,45)	39	64	350
	(15,15)	25	33	160
VHB 4910 acrylic	(300,300)	61	158	412
	(15,15)	29	40	55

High-speed electrically actuated elastomers with strain greater than 100%
Ron Pelrine, Roy Konbluh, Qibing Pei, Jose Joseph, *Science* vol. 287 (2000)

Energy Method

When charge is the free variable,
use the "internal electrical energy": $u = \frac{1}{2}\underline{D}\cdot\underline{E}$

The total electrical energy is calculated by integrating over the volume in the current placement: $\Gamma = \int_B \left\{\frac{1}{2}\underline{D}\cdot\underline{E}\right\} dV$

For a parallel-plate capacitor, $\underline{E} = (F/h)\underline{e}_3$ and $\underline{D} = (q/A)\underline{e}_3$

$$\Gamma = \int_B \left\{\frac{1}{2}\underline{D}\cdot\underline{E}\right\} dV = \frac{1}{2}\left(\frac{q}{A}\right)\left(\frac{\Phi}{h}\right)(Ah) = \frac{1}{2}q\Phi$$

Noting that F = q/C, $\Gamma = \frac{q^2}{2C}$

Referring to the "spring analogy" F ~ F, u ~ q, and $k \sim C^{-1}$, the internal electrical energy has the same form as the elastic spring energy $ku^2/2$.

As with spring energy, an alternative way to derive G for a parallel plate capacitor is by integrating F for charge increasing from 0 to q: $\Gamma = \int_0^q \frac{\hat{q}}{C} d\hat{q}$

Example

Consider the same Ogden solid as before:

$$W = \frac{\mu}{4}\left(\lambda_1^4 + \lambda_2^4 + \lambda_3^4 - 3\right) = \frac{\mu}{4}\left(\frac{2}{\lambda^2} + \lambda^4 - 3\right)$$

The total potential energy is $\Pi = WV_0 + \Gamma$

where $\Gamma = \frac{q^2}{2C} = \frac{hq^2}{2\varepsilon A} = \frac{\lambda^2 h_0 q^2}{2\varepsilon A_0}$

$$\Rightarrow \Pi = \frac{\mu}{4}\left(\frac{2}{\lambda^2} + \lambda^4 - 3\right)A_0 h_0 + \frac{\lambda^2 t_0 q^2}{2\varepsilon A_0}$$

If q is prescribed as a fixed value, then Π is only minimized w.r.t. λ:

$$\frac{d\Pi}{d\lambda} = \mu\left(\lambda^3 - \frac{1}{\lambda^3}\right)A_0 t_0 + \frac{\lambda t_0 q^2}{\varepsilon A_0} \equiv 0$$

… must solve numerically for λ.

Now suppose that Φ is prescribed/fixed and q is unknown. This is analogous to stretching a spring with a prescribed force F. In this case, the potential becomes

$$\Pi = WV_0 + \Gamma - q\Phi.$$

The proof is similar to before. Applying a voltage Φ to the capacitor results in a charge q* = CΦ at electrostatic equilibrium. Now suppose that the capacitor is loaded with additional charge δq. The electrostatic work required to elevate δq to a voltage Φ must be balanced by the change in internal electrical energy:

$$\delta\Gamma = \int_{q^*}^{q^*+\delta q} \Phi \, dq = \Phi\delta q \Rightarrow \delta\left(\Gamma - q\Phi\right) = 0$$

This implies that for variations in q, Γ must be replaced by $\hat{\Gamma} = \Gamma - q\Phi.$

Now, Π must be minimized w.r.t. λ and q:

$$\Pi = \frac{\mu}{4}\left(\frac{2}{\lambda^2} + \lambda^4 - 3\right)A_0 h_0 + \frac{\lambda^2 t_0 q^2}{2\varepsilon A_0} - q\Phi$$

$$\frac{d\Pi}{dq} = \frac{\lambda^2 t_0 q}{\varepsilon A_0} - \Phi \equiv 0 \Rightarrow q = \frac{\varepsilon A_0}{\lambda^2 t_0}\Phi$$

$$\frac{d\Pi}{d\lambda} = \mu\left(\lambda^3 - \frac{1}{\lambda^3}\right)A_0 t_0 + \frac{\lambda t_0 q^2}{\varepsilon A_0} \equiv 0 \qquad \therefore \lambda = \left\{1 - \frac{\varepsilon\Phi^2}{\mu t_0^2}\right\}^{1/6} \quad ✓$$

For cases when Φ is prescribed, a shortcut is to use the electrical enthalpy instead of internal energy. This is associated with a change of variables for Π:

$$\Pi = \Pi(\lambda, q) \rightarrow \tilde{\Pi} = \tilde{\Pi}(\lambda, \Phi).$$

Recall $\Pi = WV_0 + \Gamma - q\Phi$

$$= WV_0 - q\Phi + \int_0^q \Phi \, d\hat{q}$$

Performing an integration by parts ("Legendre Transformation")

$$\tilde{\Pi} = WV_0 - q\Phi + q\Phi - \int_0^\Phi q \, d\hat{\Phi}$$

$$= WV_0 \underbrace{- \frac{1}{2} C\Phi^2}_{\text{"Electrical Enthalpy"}}$$

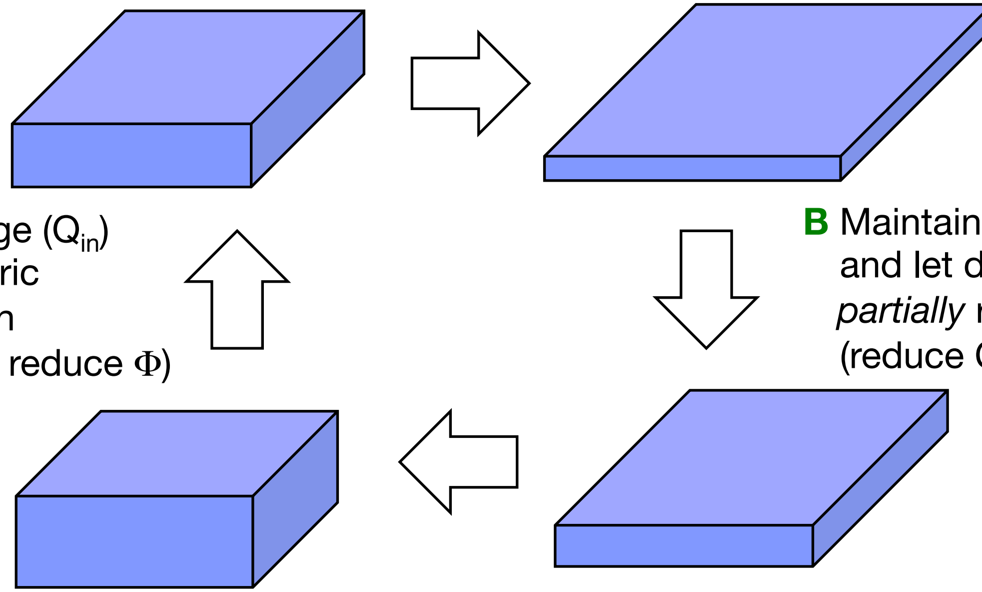
$$\Pi = \frac{\mu}{4}\left(\frac{2}{\lambda^2} + \lambda^4 - 3\right) A_0 t_0 - \frac{\varepsilon A_0 \Phi^2}{2\lambda^2 t_0}$$

$$\frac{d\Pi}{d\lambda} = \mu\left(\lambda^3 - \frac{1}{\lambda^3}\right) A_0 t_0 + \frac{\varepsilon A_0 \Phi^2}{\lambda^3 t_0} \equiv 0 \qquad \therefore \lambda = \left\{1 - \frac{\varepsilon \Phi^2}{\mu t_0^2}\right\}^{1/6}$$

DEA Generator

Dielectric starts out with a small voltage drop (Φ_{in}) and is *partially* stretched

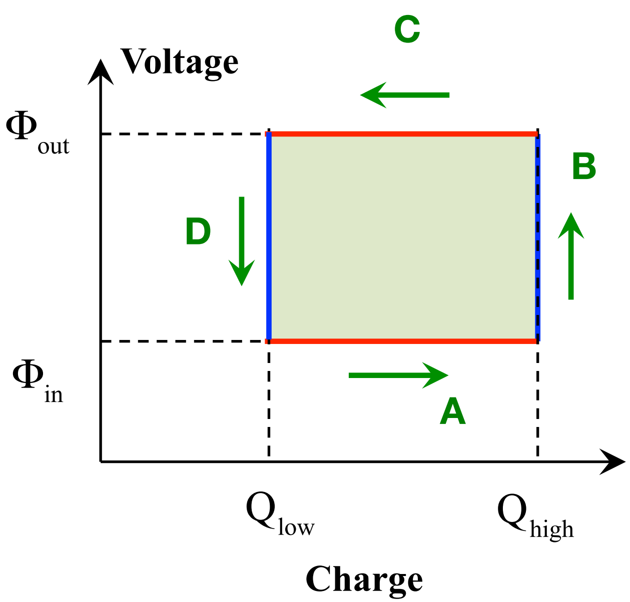
A Maintain voltage (Φ_{in}) and stretch dielectric(increase C → increase Q)
Current in (low voltage)



D Maintain charge (Q_{in}) and let dielectric *partially* stretch (increase C → reduce Φ)

B Maintain charge (Q_{high}) and let dielectric *partially* relax (reduce C → increase Φ)

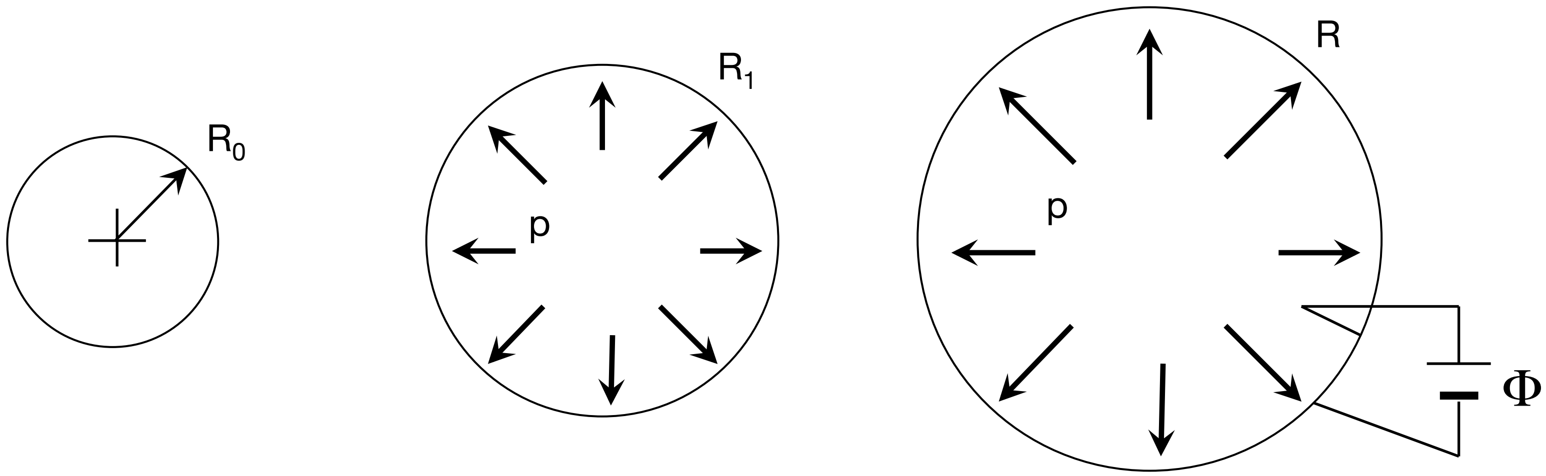
C Maintain voltage (Φ_{out}) and let dielectric *completely* relax (reduce C → decrease Q)
Current out (high voltage)



DEA Generator

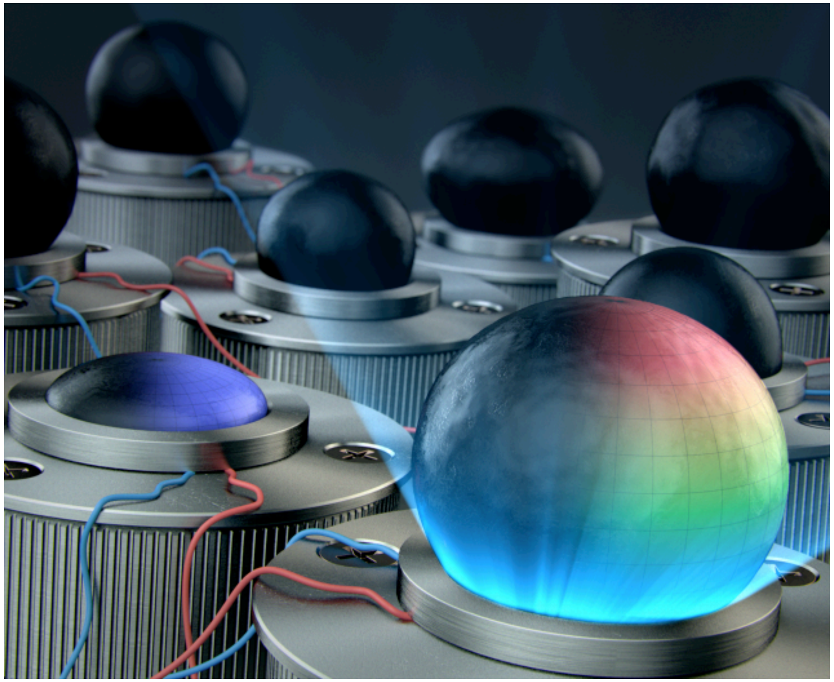
Harvest electrical energy from changes in fluid pressure

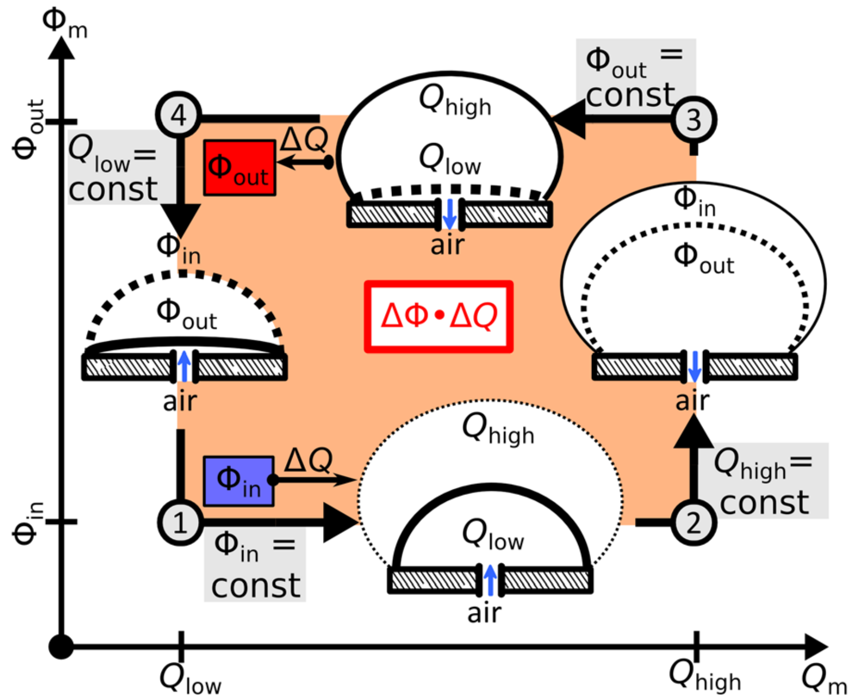
- Balloon
- Energy harvesting shoe
- Ocean waves



i Compressed Air **In** ii Voltage Applied

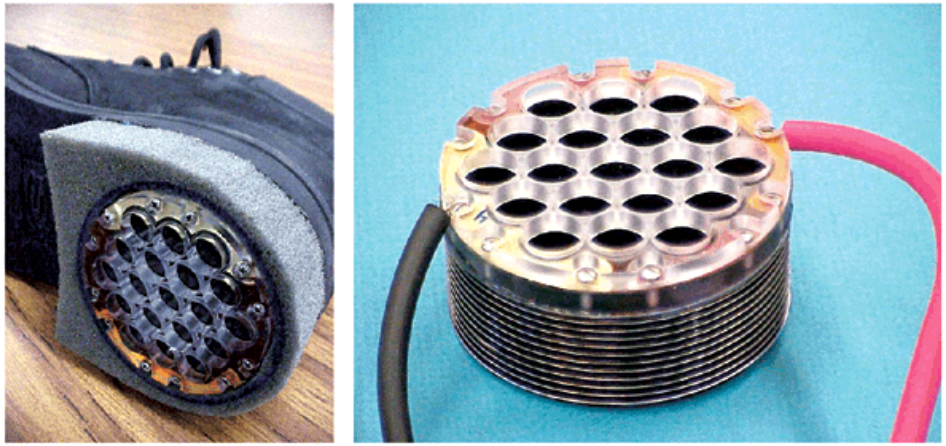
Balloon Generator



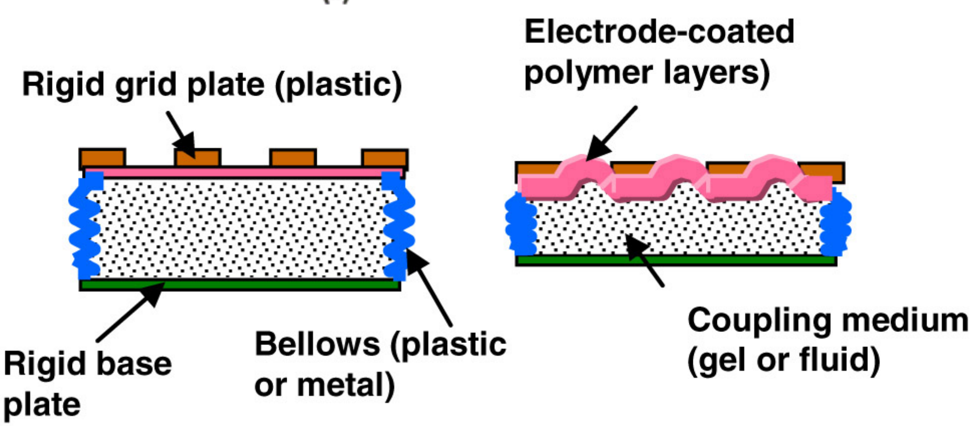


Kaltseis, Suo, Bauer, et al. *APL* (2011)

DEA Generator



(a) (b)



Iain Anderson (Univ. Auckland)

0.8 J per step
Avg. Power ~ 1 Watt
Specific Energy: 0.3J/g
33% conversion

PolyWEC

Polymeric Wave Energy Converter

- EU funded project (2012-2016; >2mil. euros)
- Lead: Marco Fontana (SSSA)
- Partner: Univ. of Edinburgh